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LINEAR PERSPECTIVE:

DESIGNED ESPECIALLY FOR THE USE OF THOSE WHO ARE PREPARING
FOR THE

SECOND GRADE EXAMINATION

OF THE

SCIENCE AND ART DEPARTMENT.

BY HENRY HODGE,
THIRD MASTER OF THE WINCHESTER TRAINING COLLEGE.

WILLIAM COLLINS, SONS, & COMPANY,
LONDON, GLASGOW, AND EDINBURGH.
1872.

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PREFACE.

THIS Book is intended as a course to be used in preparation for the Examination of the Science and Art Department in Second Grade Perspective. Only just so much theory is given as is thought necessary for the purpose, the work being intended to be mainly practical.

To aid self-instruction, all points in the working of the problems in Part II., with which it is thought the learner may find difficulty, are explained by notes below the problem.

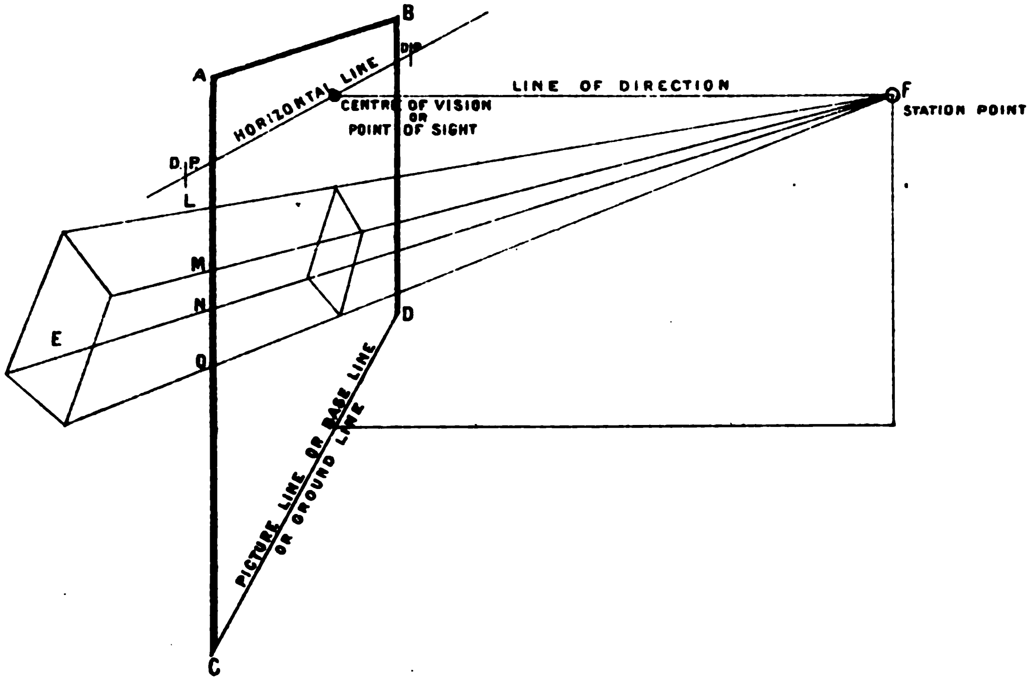
To use the book with advantage, the Student should well study and learn the laws and principles laid down in Part I., studying first and then working, as he goes on, the problems in Part II. It is not sufficient that he look over the working of the problems, but it is necessary that he should *work* them for himself as well.

H. H.

PART I.

PARALLEL PERSPECTIVE.

THE literal meaning of the word "Perspective" is "looking through," and, as applied to Drawing, means the delineation of an object on a Transparent Plane, through which it is seen. This is effected by drawing lines from every point of the object through the Transparent Plane to the spectator's eye, and then joining the points thus found on the Plane. This can be better explained by a diagram. Thus, let A B C D be a Transparent Plane, say a sheet of glass, and E, a square board standing on one of its corners behind the Plane, the object to be drawn; and let F be the eye of the spectator or draughtsman. The figure E will be drawn in Perspective on the Transparent Plane, by joining the points found on the Plane by the lines L M N O passing through it to the spectator's eye.



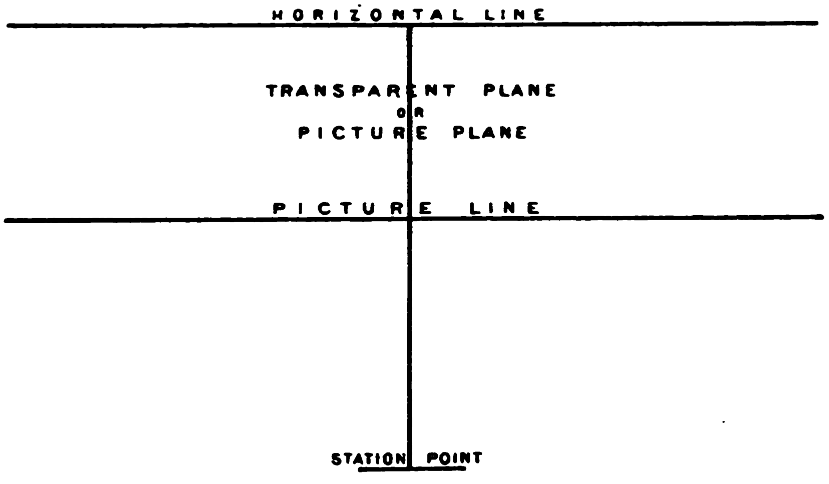
A
B
D.P.
D.P.
HORIZONTAL LINE
LINE OF DIRECTION
F
STATION POINT
CENTRE OF VISION OR POINT OF SIGHT
L
M
N
O
E
D
C
PICTURE LINE OR BASE LINE OR GROUND LINE

This, then, is really a Perspective Drawing. But it is not hard to see that this method of doing Perspective is not practicable. For instance, if I want to draw a house, it is not possible for me to have a large Transparent Plane set up before the house, and the drawing made on the Plane by the method above described.

This method, then, cannot be practised. But there have been laws and principles laid down by following which the same result can be obtained; *i.e.*, if the dimensions of an object, its distance from the Transparent Plane, its distance from the spectator, and its position be known, as correct a Perspective representation of it can be obtained as would be got by following the course above described.

To find these laws and principles, and apply them, is the object of this book.

In the diagram on the preceding page, a line will be seen running across the Transparent Plane, at a height from the ground equal to the height of the spectator's eye, called the *Horizontal Line*. It is so called because it represents the horizon of the spectator. It will also be seen that the line forming the lower boundary of the Transparent Plane, *i.e.*, the line in which the Transparent Plane and the ground meet, is called the *Picture Line*, or *Ground Line*, or *Base Line*. Another line proceeding from the spectator's eye to a point in the Horizontal Line, called the *Centre of Vision* or *Point of Sight*, and at right angles to the Transparent Plane, is called the *Line of Direction* or *Principal Visual Ray*. All three of these lines are important ones, without which no problem in Perspective can be worked, and whose names and exact positions with regard to each other must be well kept in mind. Indeed, the first thing to be done in setting about a Perspective Drawing, is to put these lines on the paper in their proper relative positions, and according to directions given. And as on a flat surface they cannot be represented as they really are, they will appear on the paper thus :—



If a problem in Perspective be given to a person to be worked, these lines will either be already drawn for him on the paper, or he will be sufficiently directed to be enabled to draw them correctly for himself. Thus, he may be told that the Horizontal Line is 5 ft. from the Ground Line, and that the Line of Direction is 10 ft. long. He could from this make the lines as required—first drawing the Horizontal Line, he would make the Picture Line 5 ft. from and parallel to it, and then draw the Line of Direction at right angles to both, and 10 ft. long.

The two points marked on the diagram, called *Distance Points*, are important ones, and are found by taking the Centre of Vision as centre, and the Line of Direction as radius, and making marks on the Horizontal Line.

Thus far we have only considered the lines necessary (as compasses, and ruler, and other instruments are necessary) to the working of problems. Let us now consider the laws and principles by which the working is to be carried on.

If railway lines, which are parallel to one another, be observed, they will be seen apparently to converge from the spectator all towards the same point. And if a person stand at one end of an avenue and look down it, the two parallel rows of trees will appear much closer together at the far end; and if the rows were long enough, they would appear quite to meet. We know these parallel lines do not meet thus, but they *appear* to meet; and all parallel lines would appear to do the same. As, then, Perspective has to do with the *appearance* of things, it is laid down as a Perspective Law that

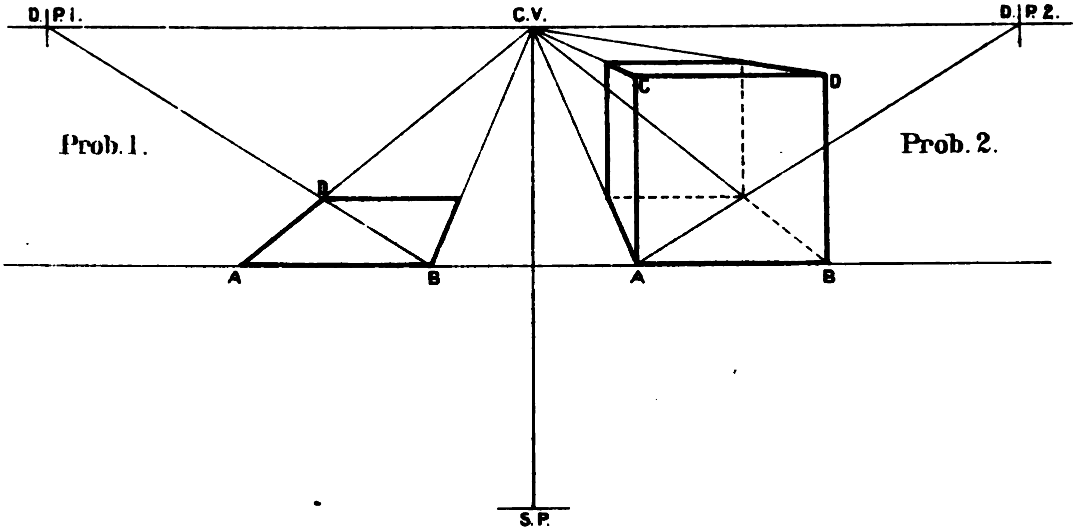
Parallel Lines vanish towards the same Point.

Again, if a person stand on the railway lines, or any set of parallel lines, and suppose a line drawn across them at right angles to them, and consider this as the Picture Line, and then observe the lines, he will see that they all appear to be running towards the point on the horizon exactly opposite his eye. And we have seen that the point on the horizon exactly opposite the spectator's eye is the Centre of Vision or Point of Sight, and the parallel lines in this case are at right angles to the Picture Line, therefore another established law in Perspective is that

Lines at right angles to the Picture Line vanish towards the Point of Sight.

We have now found out two very important laws; let us apply them by working one or two simple problems.

Problem 1.—Place in Perspective a square of 4 ft. lying on the ground, with one of its sides parallel to and touching the Transparent Plane 2 ft. to the left of the spectator—Scale, $\frac{1}{4}$ of an inch to a foot; Horizontal Line 5 ft. above the ground; Line of Direction 10 ft. long.



Now if one side of the square be parallel to the Picture Line, it is plain that there will be two of its sides at right angles to it; and these, by the law just laid down, will run to the Point of Sight. First measure 2 ft. to the left of spectator (*i.e.*, of the Line of Direction) on the Picture Line; then 4 ft. for the side of the square (A B) which is to touch the Transparent Plane. From A and B draw lines to the Point of Sight. Now comes the difficulty of the problem,—viz., to cut off 4 ft. on each of these lines. If you measure 1 ft. from A to the right on the Picture Line, and draw to Distance Point 1, you will cut off 1 ft. on the line running from A to the Centre of Vision. If you measure 2 ft. from A towards the right and draw to Distance Point 1, you cut off 2 ft. on the same line. So if you measure 4 ft. from A towards the right, and draw to the same Distance Point, you will cut off the four feet required (A D). And it will not be necessary to cut it off on the other line, for a line drawn through D parallel to A B completes the Perspective representation of the square.

And now notice that lines running to the Centre of Vision are measured by means of the Distance Points; and that the Distance Point, which is on the same side of the Line of Direction as the line which is to be measured, is generally used to measure it, although if the other be used the same result is obtained; and notice further, that in measuring on the Picture Line, you measure away from the Distance Point which you are about to use.

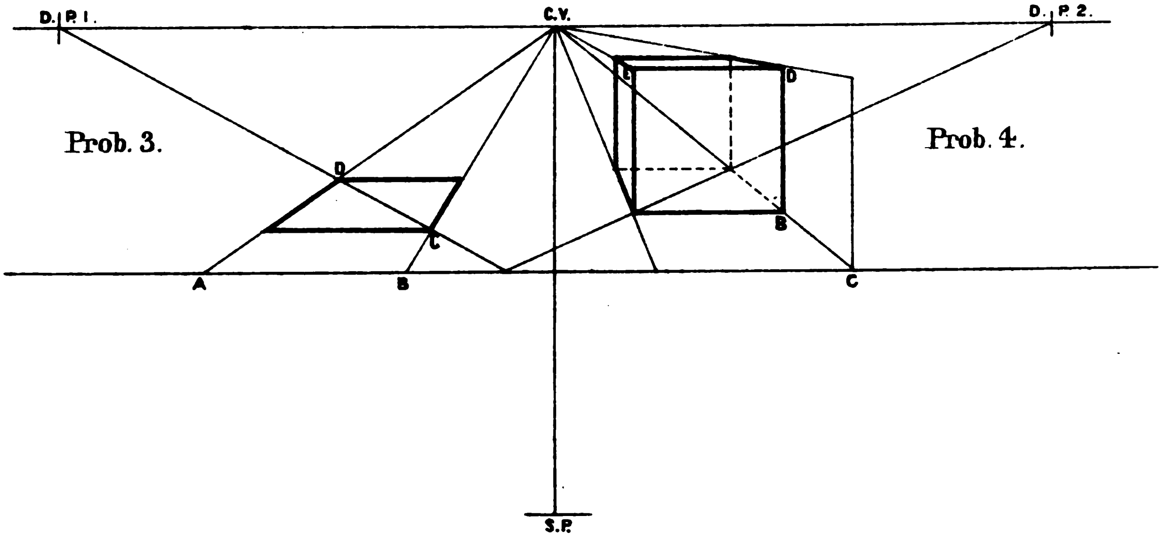
Problem 2.—Place in Perspective a cube of 4 ft. edges; the nearest face being parallel to and touching the Transparent Plane, and 2 ft. to the right of the spectator—Scale, &c., as in Problem 1.

First measure 2 ft. to the right on the Picture Line, and then 4 ft. for the front side of the base, which mark A B. From A and B draw lines to Centre of Vision, and complete the square base by Problem 1. Now set up four perpendiculars, one from each corner of the base, and make them 4 ft. high. Let us see how this is to be done.

The farther an object is away from the spectator, the smaller it appears. Everybody must have noticed that a house, for instance, close to a person appears very much larger than one the same size a long way off. This, then, is a law in Perspective,—viz., that objects appear smaller as they get farther from the spectator. But it is always supposed that no decrease in size takes place till after passing *beyond* the Transparent Plane; *i.e.*, no decrease in size is allowed for the distance between the spectator and the Transparent Plane; and lines standing on the Picture Line therefore are not diminished, because they are not *beyond* the Transparent Plane.

Since, then, the perpendiculars standing on A and B are on the Picture Line, they will be actually 4 ft. by scale high. Measure 4 ft. on each of them, and join the points C and D thus obtained, to form the front side of the square forming the top of the cube. Then run lines from C and D to Centre of Vision, and the fourth side of the top square is obtained by joining the points where these lines cut the other perpendiculars.

Problem 3.—Place in Perspective a square of 4 ft., its nearest side being 3 ft. to the left of the spectator, and 2 ft. beyond and parallel to the Transparent Plane—Scale, &c., as in Problems 1 and 2.



As before, first measure the 3 ft. left on the Picture Line, and then 4 ft. (A B). This 4 ft. is not the front side of the square, as in Problem 1, for the front side is, by the directions given, to be 2 ft. beyond the Transparent Plane; or, as is generally said, 2 ft. in the picture. Draw from A and B to the Centre of

Vision; a line drawn anywhere between these two parallel lines, parallel to the Picture Line, will be 4 ft. long. A line drawn between these two lines 2 ft. from the Transparent Plane, and parallel to it, will then be the front side of the required square. The 2 ft. in the picture will be found as 4 ft. in the picture was found in Problem 1 (for the back line of the square in that problem is 4 ft. from the Transparent Plane, or 4 ft. in the picture); *i.e.*, measure 2 ft. to the right of B, and draw to Distance Point 1. The line thus drawn gives the point C on one line 2 ft. in the picture, and the point D on the other 6 ft. in the picture; for it is drawn from a point 6 ft. to the right of the line running from A. Lines drawn through the points C and D, parallel to the Picture Line, complete the square required.

Problem 4.—Place in Perspective a cube of 4 ft. edges, its nearest face being 2 ft. to the right of the spectator, 3 ft. in the picture, and parallel to the Transparent Plane.

Make the square forming the base of this cube by Problem 3. Then set up the four perpendiculars. Now comes the difficult part of the problem,—viz., to measure 4 ft. on these perpendiculars (or on one of them, for that will be sufficient), now they stand within the picture.

To measure a height within the picture you must

(1) Find the point on the ground on which the perpendicular to be measured stands.

(2) Draw a line from *any* point in the Horizontal Line through that point to the Picture Line. (If there be a line already, by previous work, drawn from the Horizontal Line through the point to the Picture Line, that one will serve.)

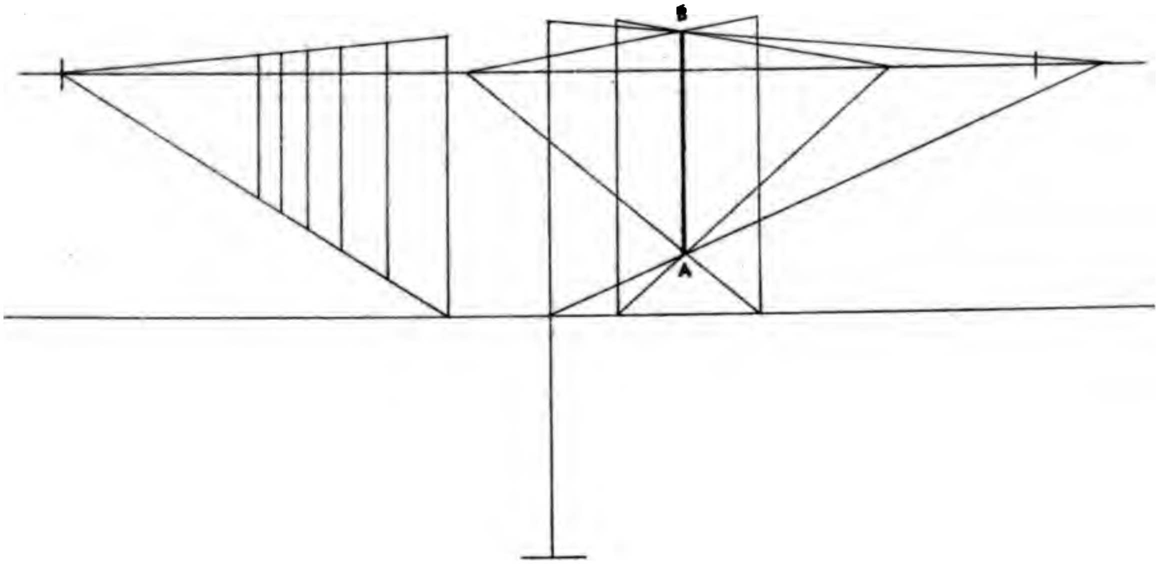
(3) On the point where this line meets the Picture Line set up a perpendicular, which you will call a Height Line.

(4) On this Height Line, from the Picture Line, mark off the height required.

(5) From the point so obtained draw back to the point in the Horizontal Line chosen in (2), and thus cut off the height on the perpendicular which is to be measured.

In the cube we are constructing, one of the perpendiculars stands on the point B, through which is a line running from the Horizontal Line and meeting the Picture Line at C. At C then set up a perpendicular or Height Line; on it measure 4 ft. from C, and draw back in this case to the Centre of Vision; the point where the line thus drawn back cuts the perpendicular standing on B will be 4 ft. from the ground. Through this point draw the line E D parallel to the Picture Line, to form the front edge of the top of the cube, and complete the figure as in Problem 2.

To make the matter of measuring heights more clearly understood, study well the following diagram,

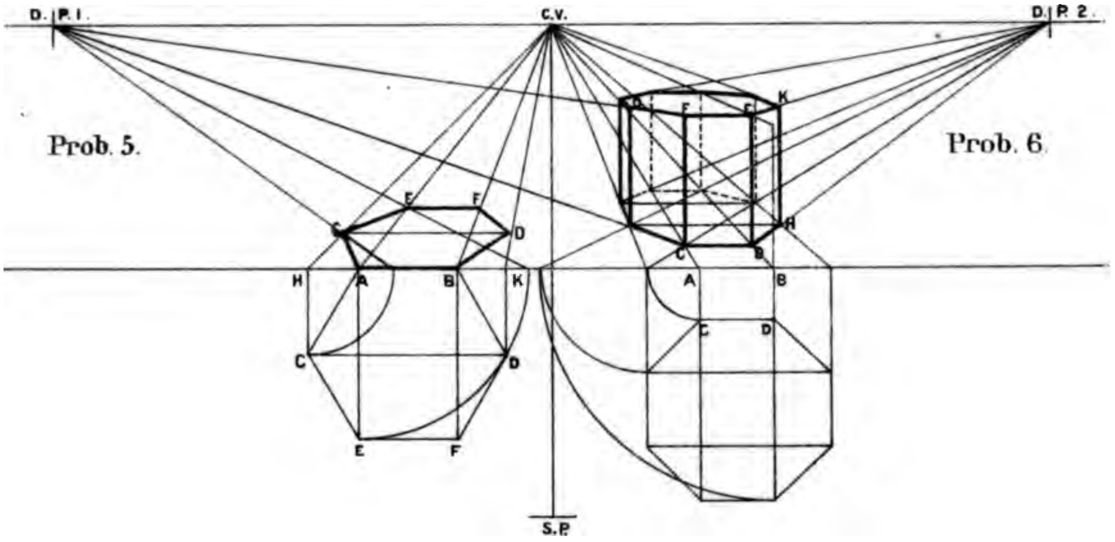


in which, on the right, the height of a line (A B) is measured several times by using several different points in the Horizontal Line, each with the same result; and on the left is a vertical plane 6 ft. high, with several lines drawn perpendicularly across it, all of which must be the same height, since the top and bottom of the plane are parallel lines vanishing to a point in the Horizontal Line.

The student may now study the working of, and then work for himself, Problems 1-13 in Part II.

Problem 5.—Place in Perspective a regular hexagon, lying on the ground, with one of its sides parallel to and touching the Transparent Plane 2 ft. to the left of the spectator, and each side being 2 ft. long.

Before commencing this problem, a geometrical plan of the hexagon must be made below the Picture Line, placed as the directions for working the problem require,—viz., with one side on the Picture Line 2 ft. to the left of the Line of Direction. Construct the plan. This is called a *Ground Plan*, and the Perspective is to be worked from it.



The side A B of the plan will likewise be a side of the Perspective representation, for one side is to touch the Transparent Plane. The point C is in a line at right angles to the Picture Line, which will vanish to the Point of Sight from the point H, where it touches the Picture Line. Draw the line to the Point of Sight, and measure on it from H a part equal to H C. This is done by measuring the length of H C, from H, on the Picture Line, towards the right, and drawing to Distance Point 1. The point C in Perspective is thus obtained. The point D is on a line at right angles to the Picture Line running from K; draw from K to Point of Sight. And as points D and C are in a line parallel to the Picture Line in the plan, they will likewise be so in Perspective. Therefore, through C in Perspective draw a line parallel to the Picture Line, and so get the point D in Perspective. The point E is in a line at right angles to the Picture Line running from A, therefore draw from A to the Point of Sight, and mark off from A, on the Picture Line to the right, the distance A E, and draw to Distance Point 1, to obtain the point E in Perspective. The point F is in a line at right angles to the Picture Line running from B, and is opposite to E in a line parallel to the Picture Line. Draw then from B to the Point of Sight, and through E draw a line parallel to the Picture Line to obtain the point F in Perspective.

The six angular points of the hexagon are now in Perspective. Join A and C, C and E, E and F, F and D, and D and B to complete the figure.

Problem 6.—Place in Perspective an octagonal prism 3 ft. high, having its nearest side parallel to the Transparent Plane, 1 ft. in the picture, and 3 ft. to the left of the spectator; and the sides of the octagonal base being each $1\frac{1}{2}$ ft. long.

A prism is a solid having *rectangular sides.* The end of a prism may be any figure; either a triangle, hexagon, octagon, or any other.

First put the octagon forming the base of the prism in Perspective. This must be done from a ground plan as in Problem 5. Make then the plan as directed; *i.e.*, having each side $1\frac{1}{2}$ ft. long, its nearest side 1 ft. from the Picture Line and parallel to it. Make this nearest side first, and on it construct the octagon.

The point C is in a line at right angles to the Picture Line which runs from A, and is 1 ft. from the Picture Line. Draw then from A to the Centre of Vision, and measure 1 ft. on this line to find the point C in Perspective. By drawing from B to the Centre of Vision, and then making a line through the Perspective point C parallel to the Picture Line, the point D in Perspective is got, and C D is the front side of the octagon in Perspective. Put in the other angular points of the octagon by Problem 5, and join them to form the figure.

On each angle of this base set up a perpendicular, and measure 3 ft. on one of them, say on the one standing on the point D. To do this follow the directions for measuring a height within the picture given under Problem 4, and set up the Height Line at B. Measure 3 ft. on the Height Line and draw back to the Centre of Vision, cutting off the required height at E. Draw through E a line parallel to the Picture Line, and the side of the top,—viz., E F, corresponding to C D of the bottom, is obtained. It is clear that each angle of the top could be found by measuring 3 ft. on each of the perpendiculars (or at least on four of them in this case), but this would be a long and clumsy method. Parallel Lines vanish to the same point in the Horizontal Line. There is to be a line running from E parallel to D H. Find then the vanishing point of the line D H by producing it to the Horizontal Line, and draw from E to this vanishing point, and so obtain the side E K corresponding to the side D H of the base. Through K draw a line parallel to the Picture Line, and this will give the point O. The other angular points of the top are found in a similar way.

The student should now study and work Problems 14-27 in Part II.

The next thing to be understood is how to put a circle into Perspective. To understand this we will work a problem.

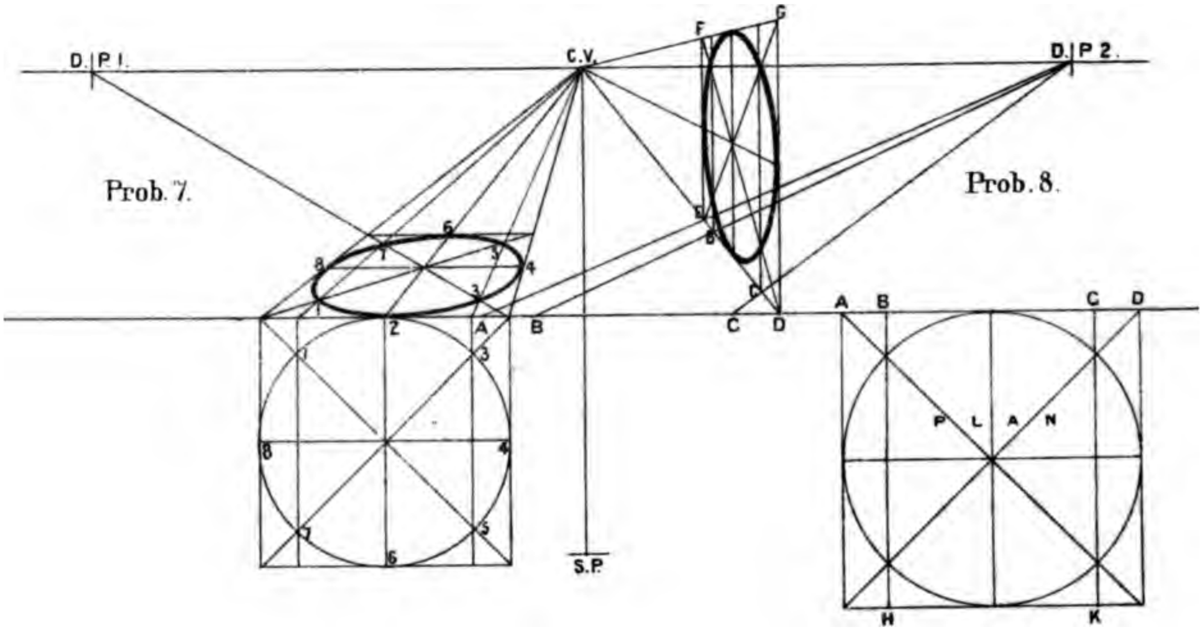
Problem 7.—Place in Perspective a circle on the ground, 5 ft. in diameter, and touching the Picture Line 4 ft. on the left of the spectator.

There is no way of putting a circle into Perspective, except by first placing a square in Perspective, and then putting the circle in the square. First, then, make a plan of the circle according to the given directions; *i.e.*, touching the Picture Line 4 ft. to the left of the Line of Direction. To do this, measure 4 ft. to the left along the Picture Line. Draw from the point thus found a line below the Picture Line, at right angles to it, and 5 ft. long. This is the diameter of the circle; describe the circle on it, and then place a square about the circle. Draw the diagonals of the square and its other diameter; and through the points where the circle and the diagonals of the square cut one another, draw lines at right angles to the Picture Line. In the square there are eight points through which the circle passes, as numbered in the Figure.

Now put the square into Perspective with its diagonals, diameters, and the lines 1-7 and 3-5 drawn in it. The circle in Perspective is then obtained by drawing the curve, by hand, through the points 1, 2, 3, 4, 5, 6, 7, 8.

We will now work another problem involving the Perspective of a circle.

Problem 8.—Place in Perspective a circle 6 ft. in diameter, in a vertical plane, which is at right angles to and touches the Transparent Plane 4 ft. to the right of the spectator—Scale, &c., as in preceding problems.



As in the last problem, the square about the circle must be placed in Perspective first—

Make a plan of the circle, place a square about it, and draw the required lines. Then measure 4 ft. to the right of the Line of Direction, and on the point thus found (D) set up a perpendicular 6 ft. high. From the top and bottom of this line draw to the Centre of Vision, because the vertical plane, in which the circle is to be made, is to be at right angles to the Transparent Plane. On the bottom line cut off 6 ft. from D, and set up another perpendicular. Thus the square D E F G, in which the circle is to be placed, is obtained in Perspective. Draw the diagonals and diameters of this square. To obtain the two lines B H and C K in Perspective, measure the distance A B in plan, and set it off on the Picture Line from A towards the right, and likewise from D towards the left, and then draw to the Distance Point 2, and so get the points B and C in Perspective; on which points set up perpendiculars which will be the lines B H and C K in Perspective. The eight required points are now found: draw the curve of the circle through them by hand.

The student should now study and work Problems 28-35 in Part II.

ANGULAR PERSPECTIVE.

TILL now we have only considered Parallel Perspective; *i.e.*, that kind in which one side at least of the figure or object is parallel to the Picture Line. But it is easily seen that an object, say a cube, can likewise be placed with regard to a line or a perpendicular plane, with one of its angles directed towards it, as well as one of its sides parallel to it. When an object is put in Perspective with one of its angles directed towards the Transparent Plane, it is in *Angular* Perspective. If a cube, for instance, be placed with one of its corners towards the Transparent Plane, it is clear that its sides will make angles with the plane, either equal, or one small and the other greater, according as the cube is placed. As yet we have only dealt with lines making angles of 90° with the Picture Plane; but if an *angle* of the cube be directed towards the plane, its sides cannot make right angles with it: they would do that only when one side of the cube is *parallel* to the plane.

We have, then, now to deal with other vanishing points than the Centre of Vision; and as an object can be placed so that its sides may make angles of *any* size with the Picture Plane, it will be necessary to know how to find vanishing points for lines making any and every angle with that plane.

In Problem 1 a square lying on the ground is placed in Parallel Perspective. If a diagonal of that square be drawn and extended to the Horizontal Line, it will be found to run to the Distance Point. Now when a square has one of its sides parallel to the Picture Plane, two of its sides make right angles with it, and therefore the diagonals will make angles of 45° with it. But we see that the diagonal of a square in Parallel Perspective runs to the Distance Point: from this we get the rule—

Lines making angles of 45° with the Picture Plane vanish towards the Distance Points.

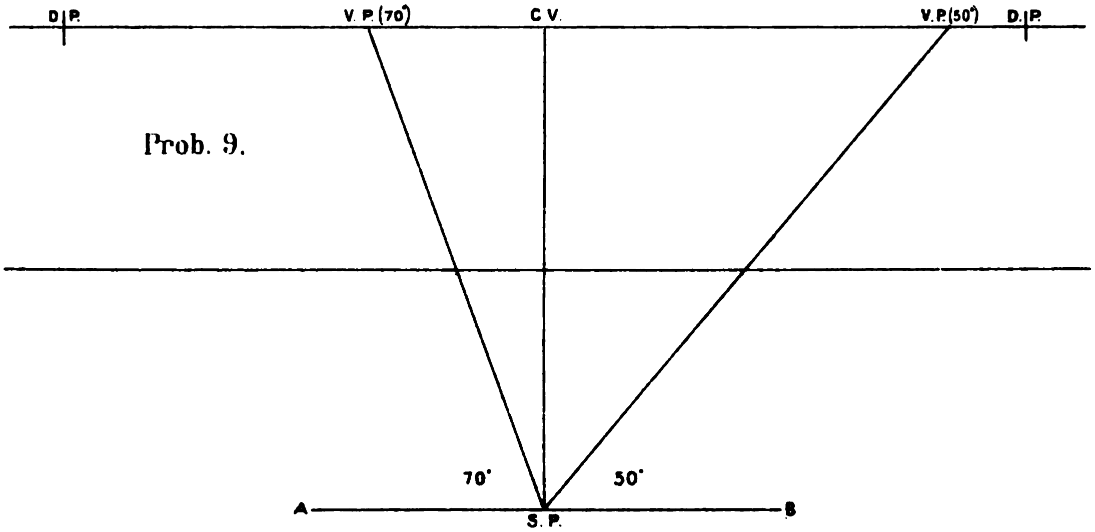
The method of obtaining the vanishing point for lines making any angle with the Picture Plane can be got from this rule. Thus: Draw a line from the Distance Point to the Station Point, and measure the angle which this line makes with a line drawn through the Station Point parallel to the Ground Line. You will find it to be an angle of 45°.

If, then, we had drawn a line through the Station Point, parallel to the Picture Line, and made an angle of 45° with that line, at the Station Point, and then produced the line forming the angle to the Horizontal Line, we should have thus found the vanishing point for lines making angles of 45° with the Picture Plane. If the vanishing point can be so found in this one case, it can be found in the same way in any other case. Thus we get the general rule for finding the vanishing points for lines making angles with the Picture Plane, viz.—

Draw a line through the Station Point parallel to the Picture Line. At the Station Point, and with this line (either right or left of the Line of Direction, according as the lines are to vanish to the right or left) make an angle equal to the angle which the vanishing lines are to make with the Picture Plane, and produce the line forming the angle to the Horizontal Line; the point at which it meets the Horizontal Line is the required Vanishing Point.

Problem 9.—Find, and mark, the vanishing point for lines vanishing to the right at angles of 50° with the Picture Plane; and likewise that for lines vanishing to the left at angles of 70° with the Picture Plane. Scale, &c., as in preceding problems.

To do this, through the Station Point draw the line A B parallel to the Picture Line. With this line, at the Station Point, make an angle of 50° on the right of the Line of Direction, and produce the line forming the angle to the Horizontal Line, to find the vanishing point for lines making angles of 50° with the Picture Plane towards the right hand. And to find the vanishing point for lines running towards the left hand at angles of 70° with the Picture Plane, make an angle of 70° with the line A B at the Station Point, on the left of the Line of Direction, and produce the line forming the angle to the Horizontal Line.



Prob. 9.

In all the problems we have worked in Parallel Perspective, distances have been cut off on lines running to the Centre of Vision (*i.e.*, making right angles with the Picture Plane) by means of the Distance Points; or, in other words, they have been *measured* by means of the Distance Points; or, the Distance Points have been the *measuring points* for the Centre of Vision as Vanishing Point.

Now as the Centre of Vision, when it is Vanishing Point, has its *special* Measuring Point, so *every* Vanishing Point has its special Measuring Point. That

Every Vanishing Point has its own Special Measuring Point

must be well kept it mind.

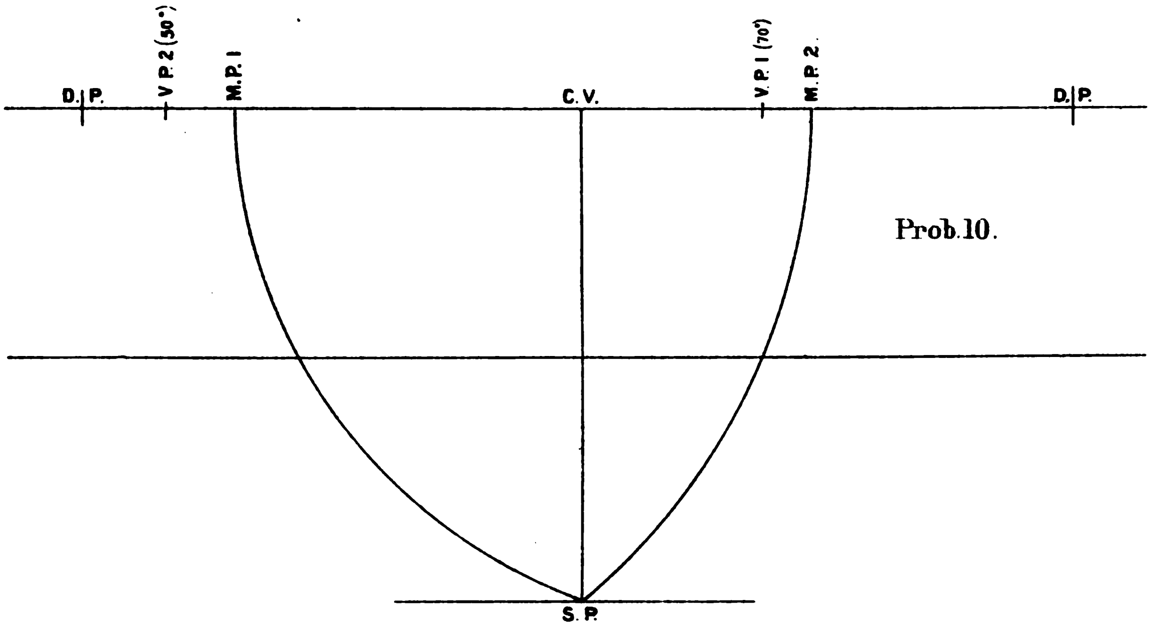
And the measuring point for any vanishing point is always found in precisely the same way as the measuring point for the Centre of Vision (*i.e.*, the Distance Point) is found.

We have seen that to find the Distance Points (*i.e.*, the measuring points for the Centre of Vision as Vanishing Point), we measure from the Centre of Vision (*i.e.*, the vanishing point) to the Station Point, and set off this distance on the Horizontal Line. The rule then for finding the measuring point for any vanishing point is—

Measure from the Vanishing Point to the Station Point, and set off on the Horizontal Line from the Vanishing Point towards the Line of Direction.

It will be manifest that by following this rule the vanishing point will always be on one side of the Line of Direction, and its measuring point on the other side.

Problem 10.—The vanishing point for lines making angles of 50° with the Picture Plane towards the left, and likewise that for lines making angles of 70° with the Picture Plane towards the right, are given. Find, and mark, their respective measuring points.



Prob.10.

Follow the rule; *i.e.*, measure from vanishing point to the Station Point and set off from Vanishing Point 1 on the Horizontal Line to find Measuring Point 1. And measure from Vanishing Point 2 to the Station Point, and set off from Vanishing Point 2 on the Horizontal Line, to find Measuring Point 2.

Observe that the Vanishing Point and its Measuring Point are on opposite sides of the Line of Direction.

Problem 11.—Place in Perspective a square of 5 ft. lying on the ground, its sides making equal angles with the Picture Plane, and the nearest angle being 3 ft. to the left of the spectator, and touching the Picture Plane.

If the sides of a square make equal angles with the Picture Plane, it is evident that these equal angles are angles of 45°.

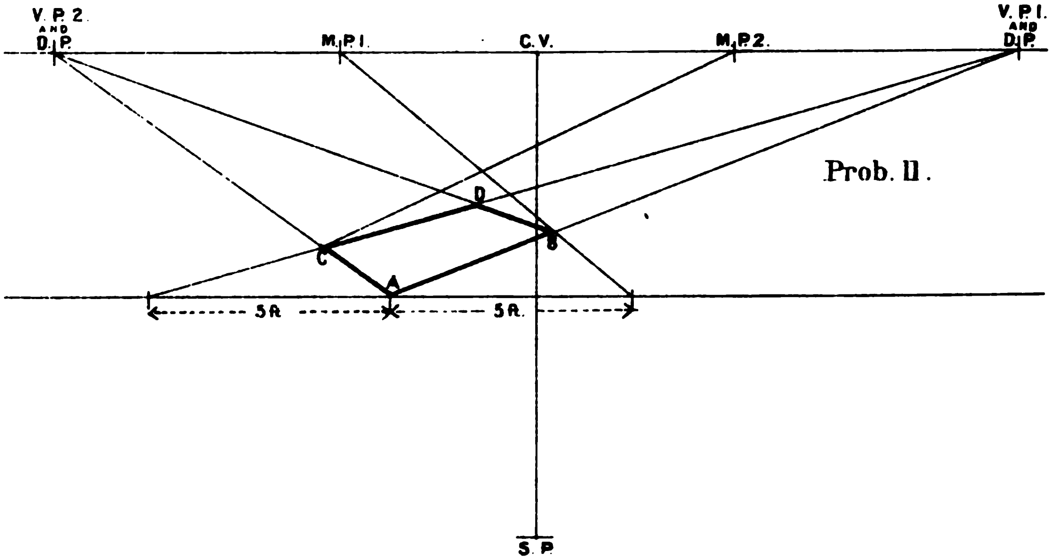
Lines making angles of 45° with the Picture Plane, vanish towards the Distance Points. The Distance Points then are the vanishing points in this problem.

Find a point 3 ft to the left of the spectator on the Picture Line. Draw from this point to each Distance Point. 5 ft. cut off on each of these lines will give two sides of the required square.

The *measuring* points are those by which distances on vanishing lines are measured. These two vanishing lines, then, cannot be measured till we have found the measuring points. Find them by the given rule; then measure 5 ft. to the right from A and draw to Measuring Point 1, and so cut off 5 ft. at the point B on the line running from A to Vanishing Point 1. Measure 5 ft. from A to the left and draw to Measuring Point 2, and thus cut off 5 ft. at the point C on the line running from A to Vanishing Point 2. A C and B C are then sides of the required square.

From B will run a side parallel to A C, and from C will run a side parallel to A B. Parallel lines vanish towards the same point, therefore from B draw to Vanishing Point 2, and from C draw to Vanishing Point 1.

These lines cut one another in the point D, which is the fourth angle of the square.



Observe that the lines running to Vanishing Point 1 are measured by Measuring Point 1, and those running to Vanishing Point 2 are measured by Measuring Point 2.

Always in lettering and figuring, put the same number to the Vanishing Point and its corresponding Measuring Point: it will save confusion.

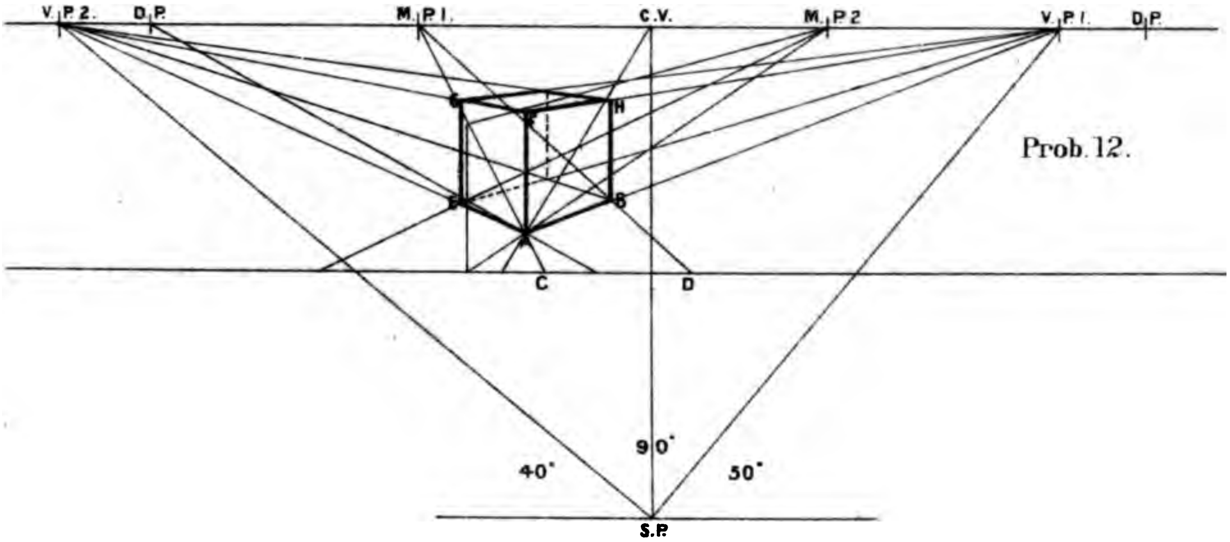
Problem 12.—Place in Perspective a cube of 3 ft. edges, one of its sides making angles of 50° with the Picture Plane towards the right, and its nearest angle being 3 ft. to the left of the spectator, and 2 ft. in the picture.

First find the vanishing points and measuring points by the given rules. If one side of the cube makes an angle of 50° with the Picture Plane towards the right, it is evident that one of its sides will make an angle of 40° with it towards the left.

Always in placing a solid or any object in Perspective, put in the base first. In this case the base is a square of 3 ft.; put this in first, as the directions require,—viz., 3 ft. to the left of the spectator, and 2 ft. in the picture. Measure 3 ft. to the left on the Picture Line, and from the point thus found, draw to the Centre of Vision. Cut off 2 ft. on this line at A. The point A is 3 ft. to the left and 2 ft. in the picture.

And now notice that *distance in the picture* is always found by using the Centre of Vision and Distance Point. Vanishing points and measuring points have nothing to do with distance in the picture.

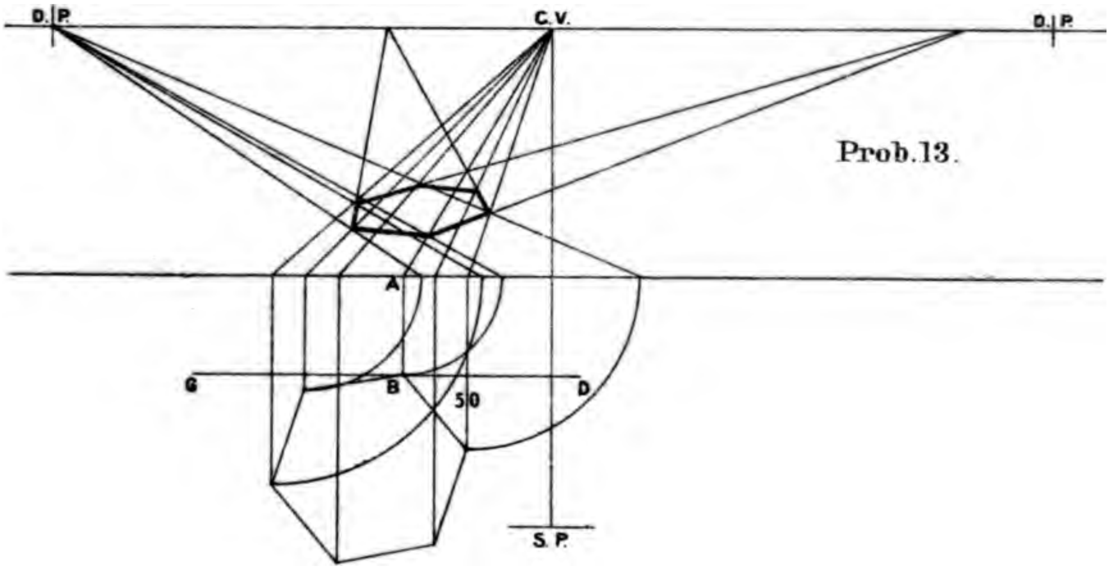
From the point A draw a line to each vanishing point, and cut off, from A, 3 ft. on each of these lines, to form two sides of the square base. Draw from Measuring Point 1 through A to the Picture Line. From the point where this line meets the Picture Line, set off 3 ft. towards the right, and draw back to Measuring Point 1, thus cutting off 3 ft. at B on the line running from A to Vanishing Point 1. All the lines making the same angle with the Picture Plane, and lying between the lines running from C and D to Measuring Point 1 are the same length, viz., 3 ft. Now draw from Measuring Point 2, through A to the Picture Line; and from the point where this line meets the Picture Line, mark off 3 ft. to the left, and draw back to Measuring Point 2, so cutting off 3 ft. at the point E on the line running from A to Vanishing Point 2. Finish the base by drawing from B to Vanishing Point 2, and from E to Vanishing Point 1.



Set up a perpendicular from each of the four angles of the base, and measure on one of these, say the one standing on A, the required height,—viz., 3 ft. There is a line running through A from Measuring Point 2: where this line meets the Picture Line set up a Height Line, and measure on it 3 ft. from the Picture Line, and draw back to Measuring Point 2: 3 ft. is thus cut off at the point F on the perpendicular standing on A. F will be the angle of the square forming the top corresponding to the angle A of the base. From F will run a line parallel to A B (*i.e.*, running to Vanishing Point 1), and likewise one parallel to A E (*i.e.*, running to Vanishing Point 2). These two lines, cutting two of the perpendiculars, give two other angles of the top square, viz., G and H. From G draw to Vanishing Point 1, and from H draw to Vanishing Point 2, to complete the figure.

Problem 13.—Place in Perspective a regular hexagon lying on the ground, each side being 2 ft. long; with one side running towards the right at an angle of 50°, and the nearest angle 3 ft. to the left of the spectator, and 2 ft. in the picture.

As has been before remarked, a polygon can only be placed in Perspective from a Ground Plan.



First, then, make the plan of the hexagon, with its nearest angle 3 ft. to the left, and 2 ft. from the Picture Line, and one side running towards the right at an angle of 50° with the Picture Plane. To do this, from a point on the Picture Line 3 ft. to the left, drop a line perpendicular to the Picture Line, and 2 ft. long, as A B. Through B draw a line C D parallel to the Picture Line. From B draw a line 2 ft. long, towards the right, below C D, making an angle of 50° with C D. This line is one side of the hexagon in plan. On it construct the hexagon, and put each of its angular points into Perspective by the method followed in Problems 5 and 6. Join these points to complete the hexagon in Perspective.

The student can now study and work the remaining problems in Part II. And he should be careful always to keep his work neat and clean. A good plan, and one which adds much to the appearance of the Perspective Drawing, is to make the lines of the object which are visible blacker and firmer than the mere working lines, and to dot the lines of the object which are not seen. Neatness in Perspective is a very important thing.

All the problems in Part II. are worked to
Scale—⅓ in. to a ft.
Length of Line of Direction—11 ft. (except where otherwise stated.)
Distance between Horizontal and Base Lines—5 ft.

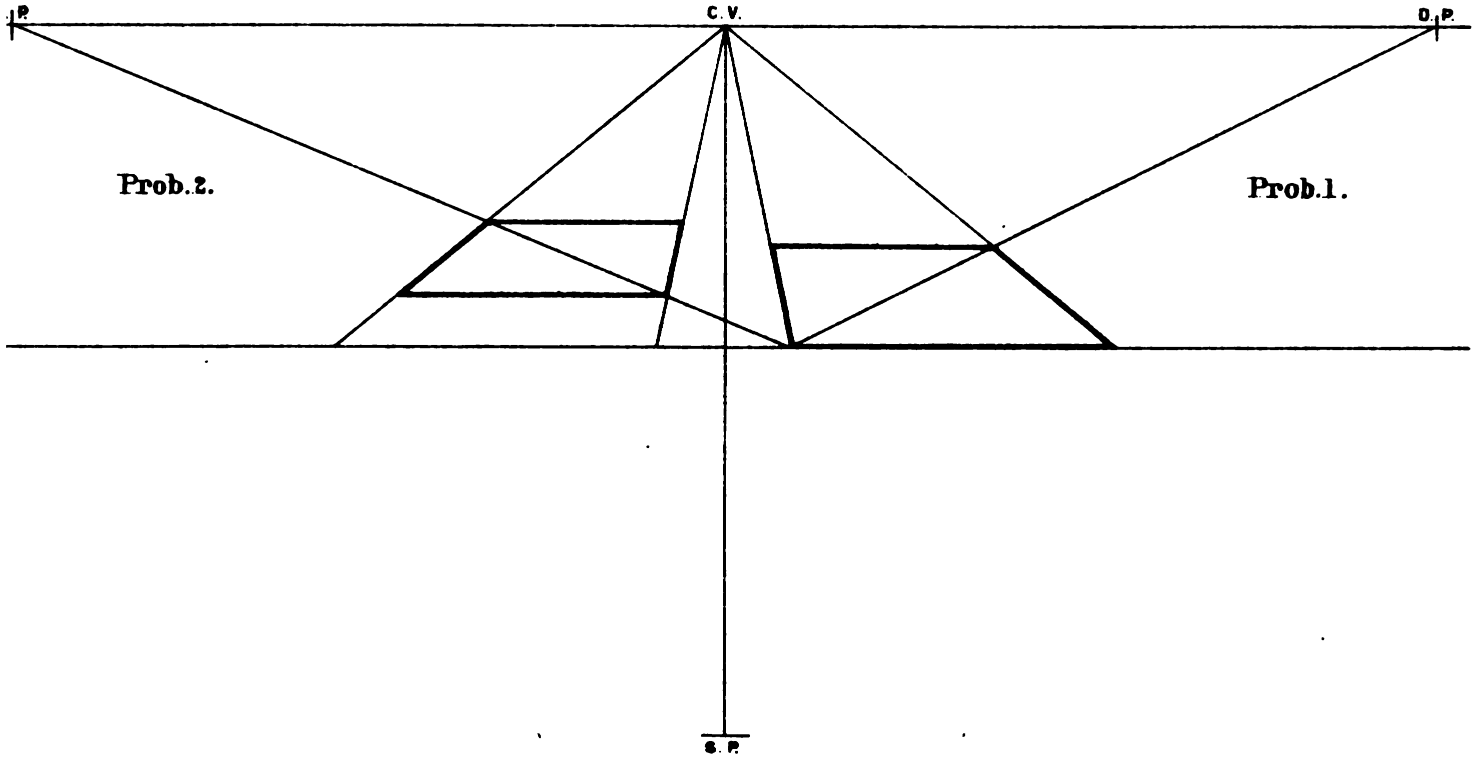
As the Science and Art Department on their Examination Papers generally ask for, besides the solution of Problems, one or two Definitions, those following should be carefully read :—

DEFINITIONS.

1. **The Centre of Vision or Point of Sight** is that point on the Picture Plane exactly opposite the spectator's eye.
2. **The Line of Direction or Principal Visual Ray** is a line supposed to proceed from the eye of the spectator to the Centre of Vision. It is at right angles to the Picture Plane.
3. **The Picture Plane, or Transparent Plane, or Plane of Delineation** is the Vertical Plane which is always supposed to be placed before the object to be represented, and on which the drawing is supposed to be made.
4. **The Ground Line, Picture Line, or Base Line** is the line forming the lower edge of the Picture Plane, *i.e.*, the line in which the Picture Plane touches the ground.
5. **The Distance Points** are two points on the Horizontal Line found by setting off from the Point of Sight, on either side of it, the length of the Line of Direction.
6. **The Station Point** is the place of the spectator.
7. **The Horizontal Line** (so called because it represents the horizon) is a line of unlimited length drawn across the Picture Plane, through the Centre of Vision, parallel to the Base Line.
8. **A Vanishing Point** is a point in which parallel lines appear to meet.
9. **Accidental Vanishing Points** are those which do not fall on the Horizontal Line.
10. **The Ground Plane** is the plane on which the object to be drawn stands.

PART II.

Problems 1 and 2.—These will only want looking at in order to be understood, if the student has mastered Part I.



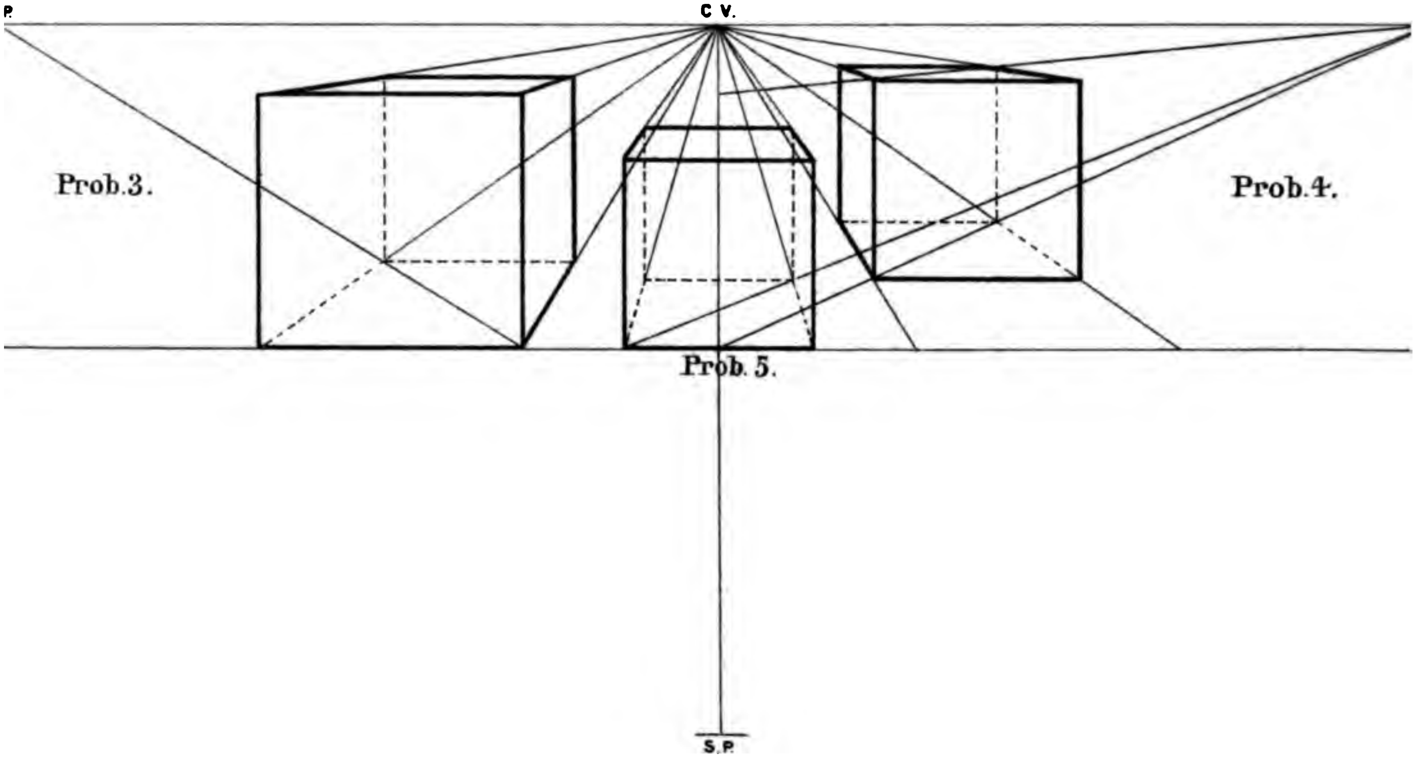
2. Place in Perspective a square of 5 ft., its nearest side being 3 ft. in the picture and 3 ft. to the left of the spectator.

1. Place in Perspective a horizontal square of 5 ft. lying on the ground with the front side touching the Picture Line 2 ft. to the right of the spectator.

Problem 3.—This is so much like Problem 2 in Part I. that there will be no difficulty with it.

Problem 4.—The height in this problem is measured on A B. Part of the Line of Direction is used as Height Line, because, as placed on the paper, it happens to stand at the point where a line running from the Distance Point through B (the point on the ground on which the perpendicular to be measured stands) meets the Picture Line.

Problem 5.—In this case the cube is standing exactly in front of the spectator, so that only one side and the top is seen.

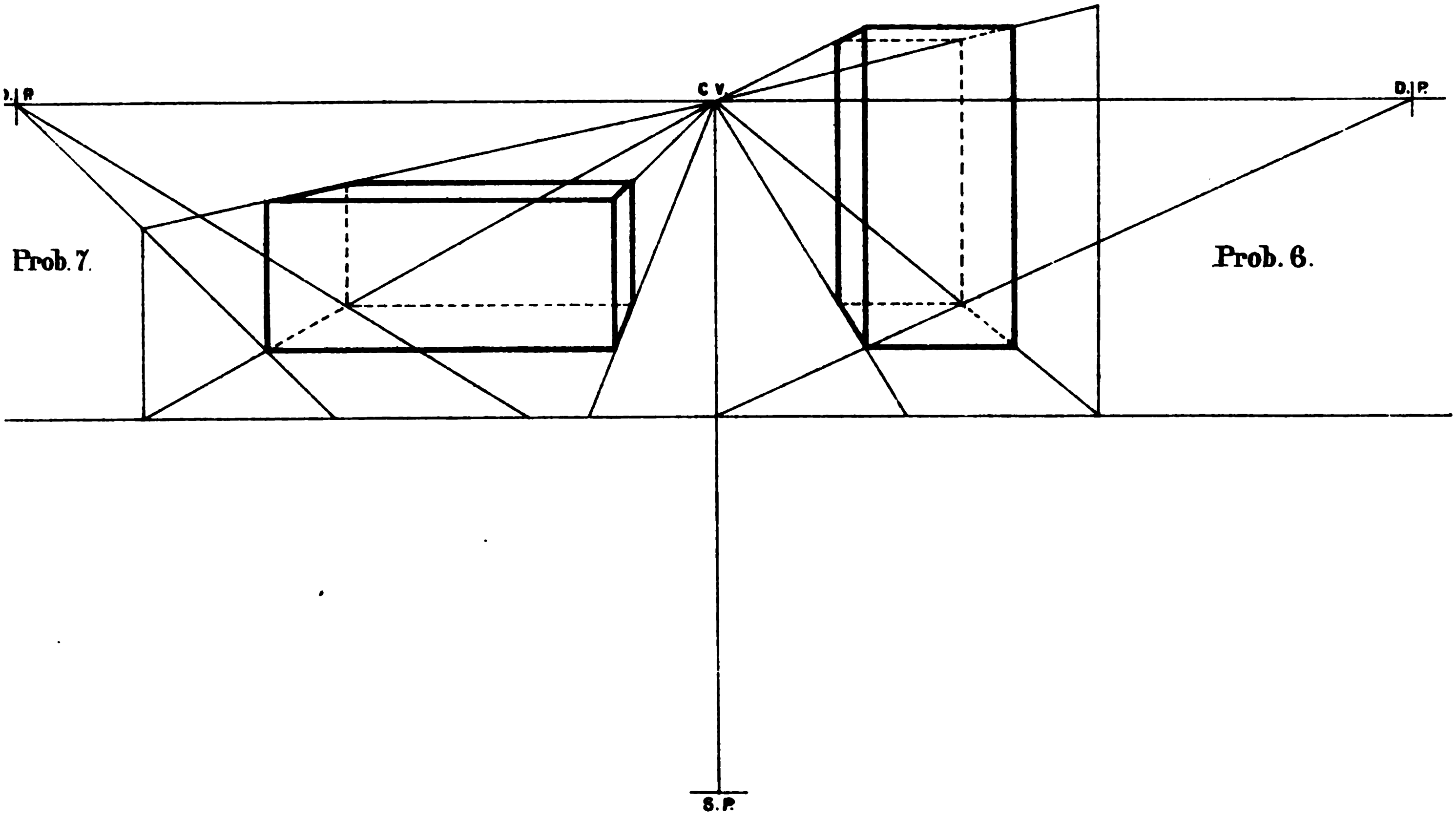


3. Place in Perspective a cube of 4 ft. edges, its nearest face being parallel to and touching the Transparent Plane 3 ft. to the left.

4. Place in Perspective a cube of 4 ft. edges, its nearest face being parallel to the Transparent Plane 3 ft. in the picture and 3 ft. to the right.

5. Place in Perspective a cube of 3 ft. edges, its nearest face being parallel to and touching the Transparent Plane immediately in front of the spectator.

Problems 6 and 7.—The working of these is so similar to the working of Problems 3, 4, and 5 that they will be easily accomplished.



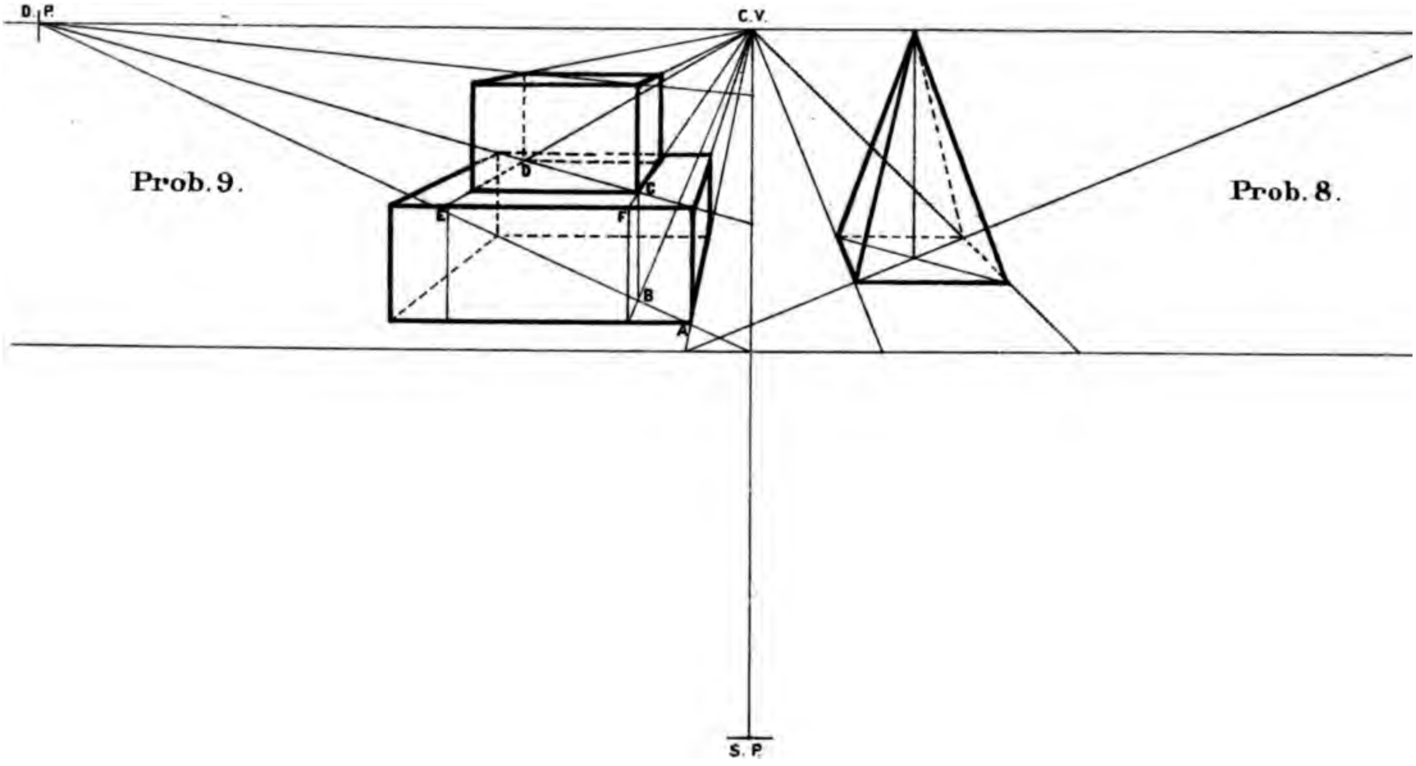
7. The same solid lying on one of its sides, 2 ft. to the right and 3 ft. in the picture, its ends making right angles with the Picture Plane.

6. A rectangular solid 6½ ft. high standing on one of its ends, which are 3 ft. square, 3 ft. to the right, and 3 ft. in the picture, and having one side parallel to the Picture Plane.

Problem 8.—The height in this problem must of course be measured on a line standing on the centre of the base, since the apex of a pyramid is exactly over that point; and as the horizontal line is 5 ft. from the Picture Line, and the pyramid is to be 5 ft. high, the apex will just reach to the horizon, and no Height Line will be required.

Problem 9.—The height of both slabs is measured by the same Height Line, since the line running from the Distance Point through A likewise goes through B.

The points C and D are found by the diagonal of the square cutting the lines running from E and F to the Centre of Vision.



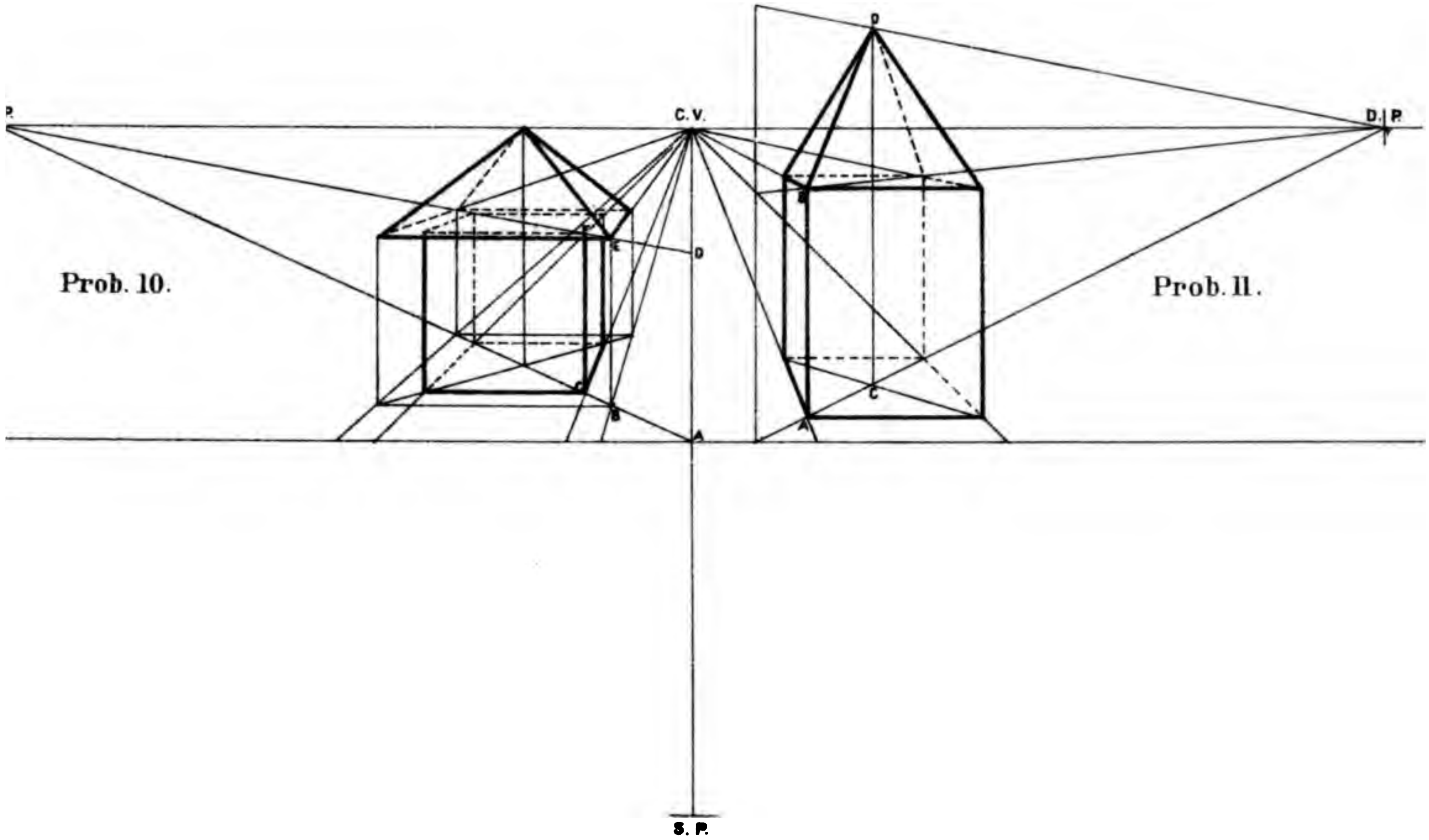
9. A stone slab 5 ft. square and 2 ft. thick, 1 ft. to the left and 1 ft. in the picture; on the ground with one side parallel to the Picture Plane. Centrally on it another slab 3 ft. square and 1 ft. thick.

8. A pyramid on a base 3 ft. square, 3 ft. in the picture, and 2 ft. to the right, and 5 ft. high.

Problem 10.—The line running from A to the Distance Point forms the diagonal of both squares on the ground, and so gives the points B and C; and the line running from D to the Distance Point forms the diagonal of both squares (viz., that forming the top of cube and the one forming the base of the pyramid), and so gives the points E and F.

The cube being 3 ft. high, and the pyramid 2 ft., the apex of the latter will fall on the Horizontal Line, since that line is 5 ft. from the ground.

Problem 11.—The same Height Line is used to measure both the lines A B and C D, since a line from the Distance Point runs through both A and C.

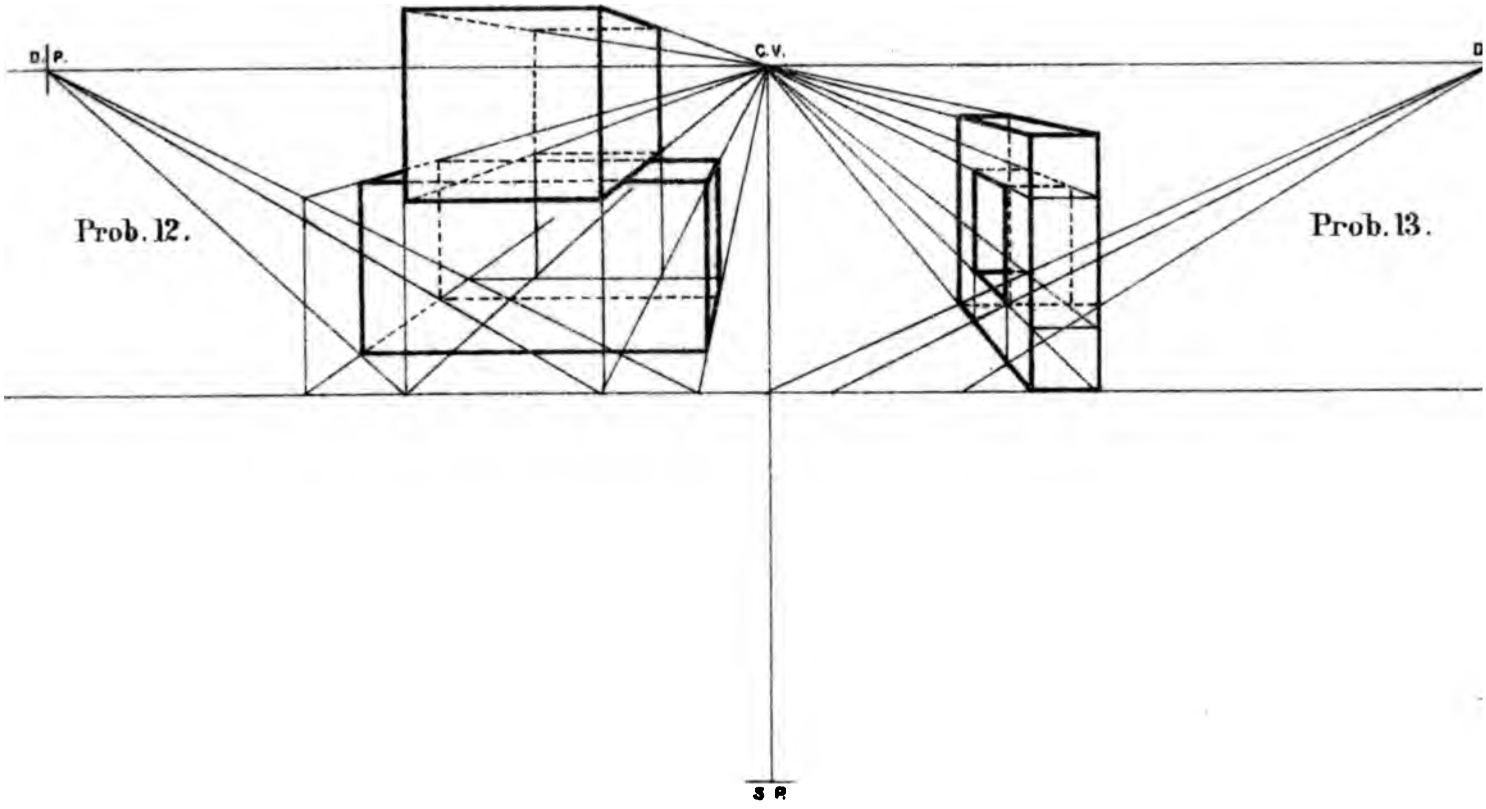


10. A cube of 3 ft. edges 2 ft. to the left and 2 ft. in the picture; and place centrally on it a rectangular pyramid 2 ft. high, with base 4 ft. square.

11. A rectangular solid on a base 3 ft. square, 2 ft. to the right, and 1 ft. in the picture; and place centrally on it a rectangular pyramid 3 ft. high, on a base 3 ft. square, whose sides are parallel to the sides of the solid. Height of solid 4 ft.

Problem 12.—The lower block being 1½ ft. in the picture, it is plain that the end of the upper block will touch the Picture Plane, for it will extend 1½ ft. on each side of the lower block. And the end as it touches the Picture Plane, will be measured by actual measurement.

Problem 13.—This requires care, but it is not difficult.



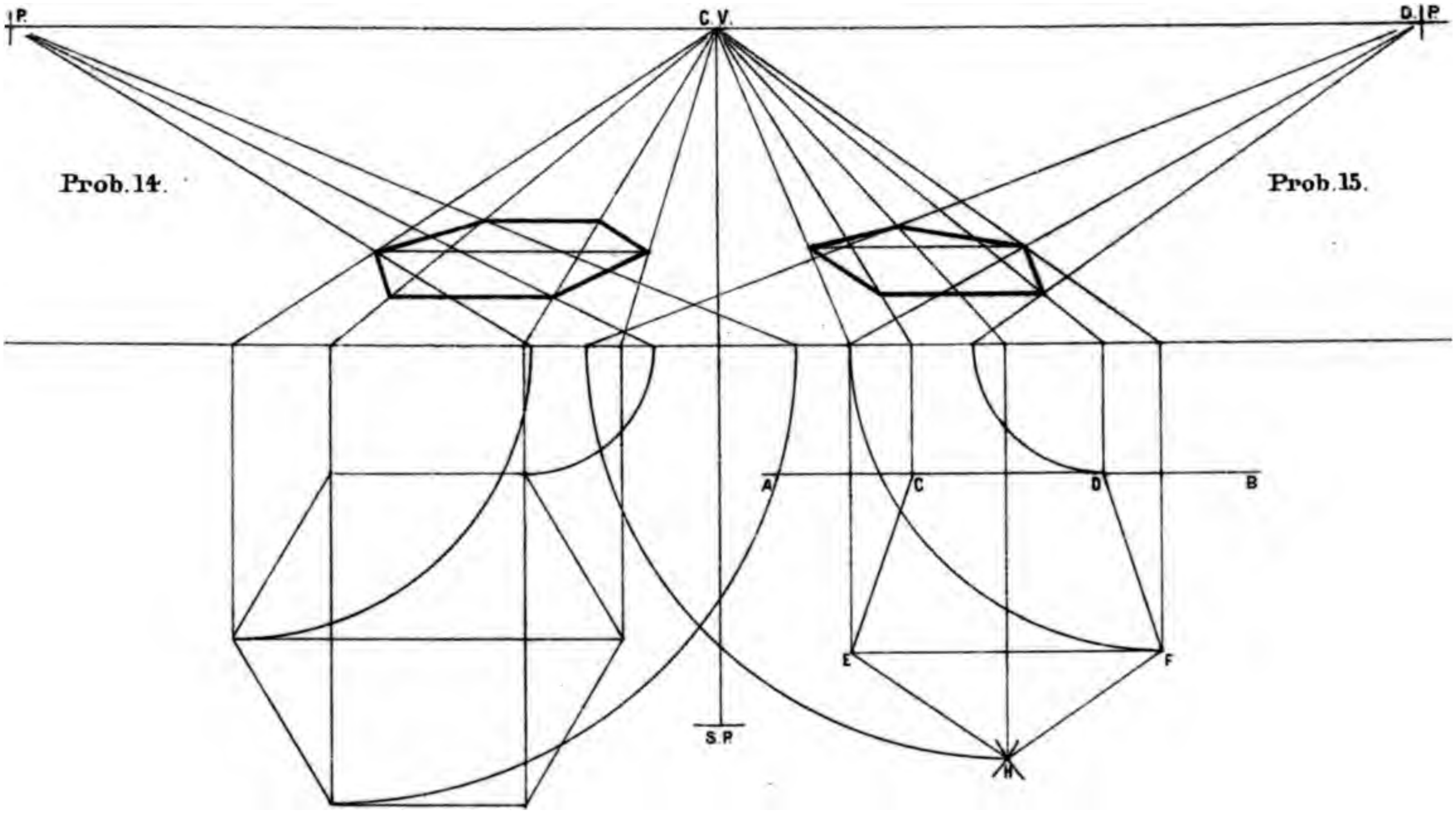
12. A block of stone 6 ft. long and 3 ft. square, lying on one of its long sides, which are parallel to the Picture Plane, 1 ft. to the left and 1½ ft. in the picture; and on the top of it place another block, the same size, with its long sides making right angles with the Picture Plane, and its centre perpendicularly over the centre of the lower block.

13. A square frame made of bars 1 ft. square. The outside edges of the four sides to be 4 ft. long. The frame to make right angles with the Picture Plane, and one bar to touch it 4 ft. to the right.

Problems 14 and 15.—There is nothing in these two problems which will be difficult to one who has well mastered what is said about placing polygons in Perspective in Part I.

The following is an easy way of making the plan of the pentagon :—

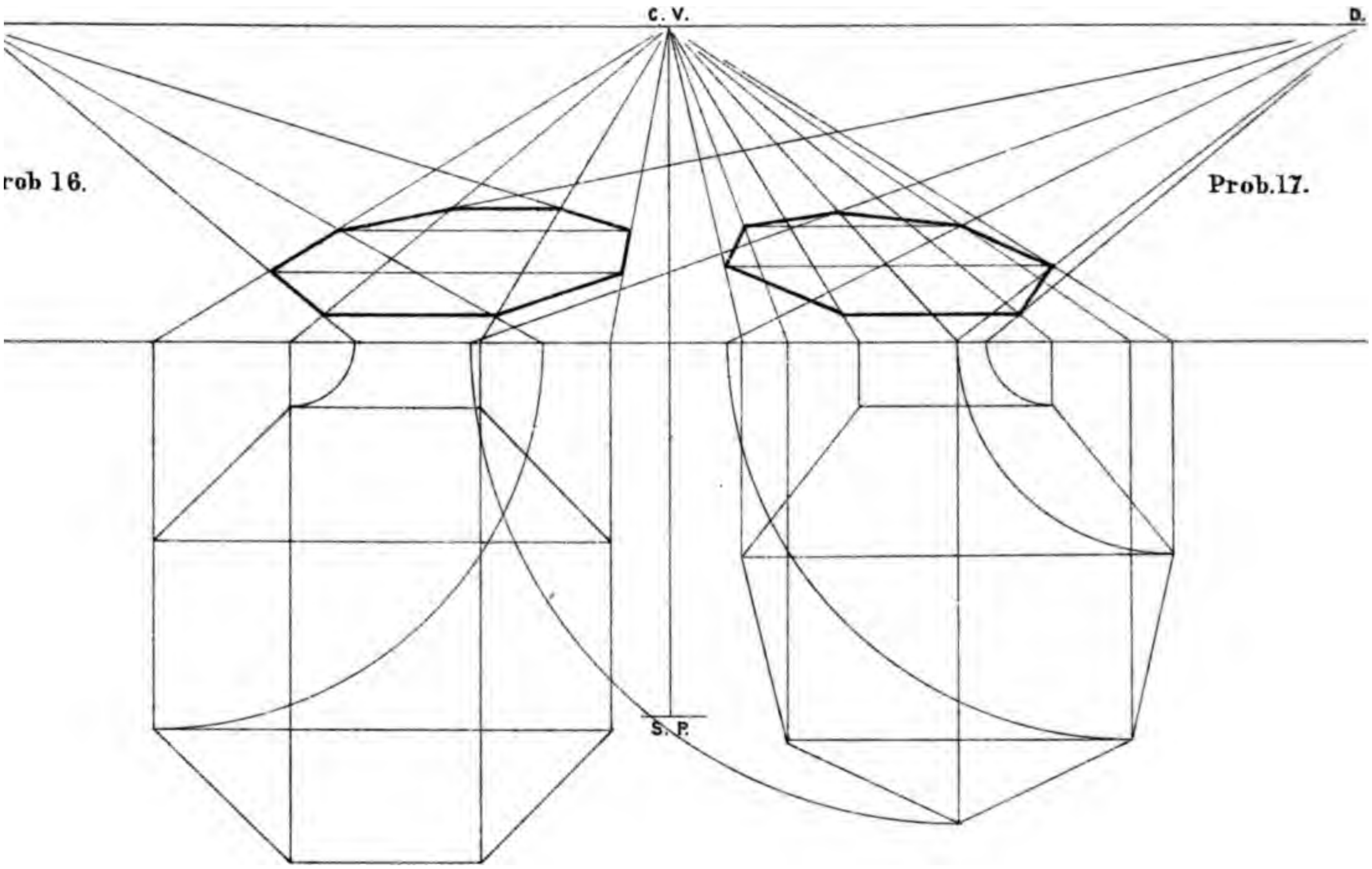
At the points C and D make angles of 72° with the line A B. Set off on the lines forming these angles the length of the side of the pentagon (in this case 3 ft.) at E and F. Take E and F as centres and the side of the pentagon as radius, and describe arcs cutting each other in H. Join E H and F H.



14. A regular hexagon of 3 ft. sides lying on the ground, with its nearest side 3 ft. to the left, 2 ft. in the picture, and parallel to the Picture Plane.

15. A regular pentagon of 3 ft. sides lying on the ground, with its nearest side 3 ft. to the right, 2 ft. in the picture, and parallel to the Picture Plane.

Problems 16 and 17.—If what is said in Part I. about putting polygons into Perspective be well known, the working of these problems will be easily understood.

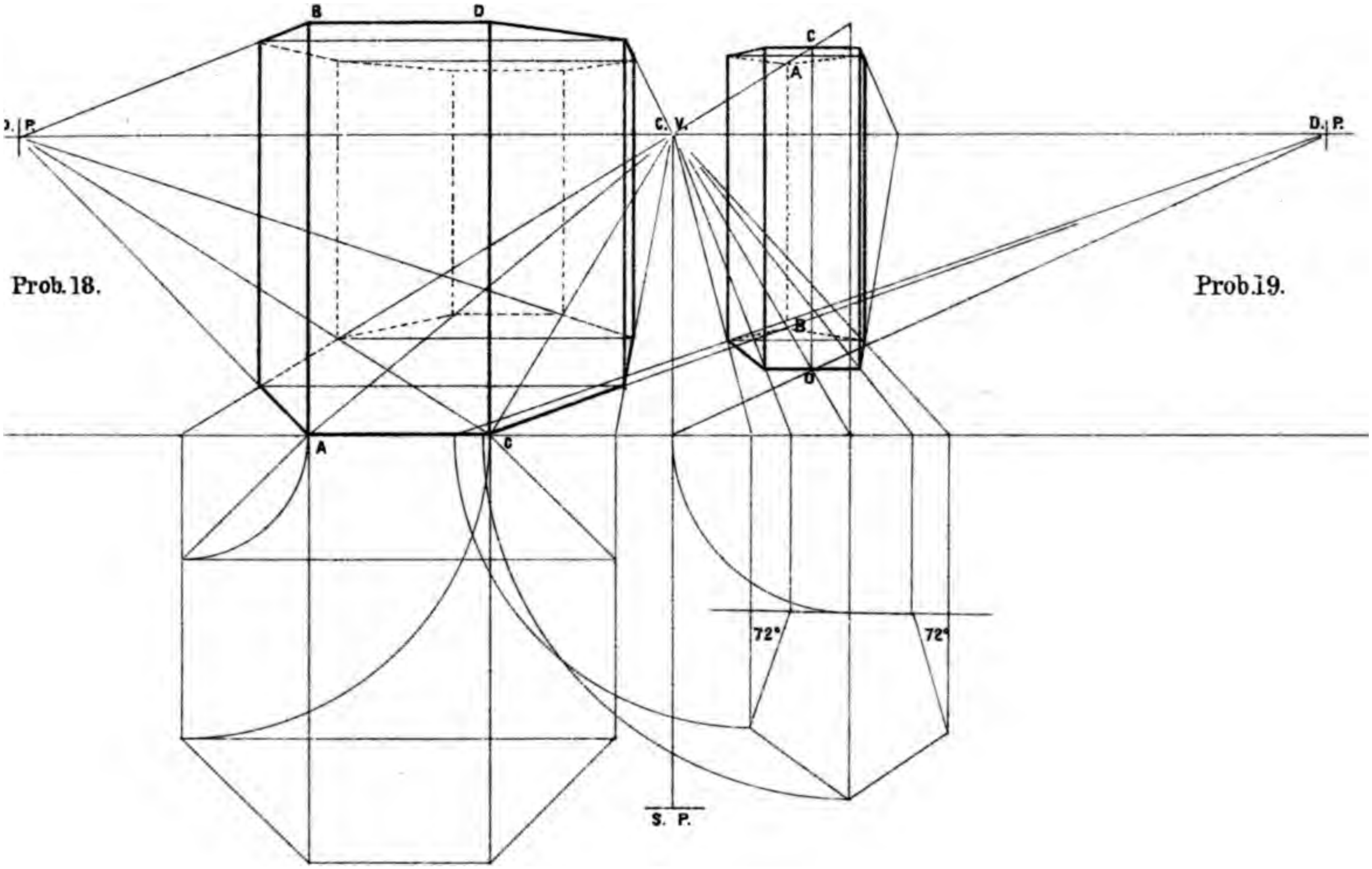


16. A regular octagon of 3 ft. sides lying on the ground—nearest side 3 ft. to the left, 1 ft. in the picture, and parallel to the Picture Plane.

17. A regular heptagon of 3 ft. sides lying on the ground—nearest side 3 ft. to the left, 1 ft. in the picture, and parallel to the Picture Plane.

Problem 18.—The lines A B and C D standing on the Picture Line are measured by actual measurement.

Problem 19.—The height is here measured on A B. The front side of the top is drawn through the point C, where the line running from the Centre of Vision through A cuts the perpendicular standing on D, because the point D is at the intersection of a line running from the Centre of Vision and the same perpendicular, C being the point of the top corresponding to D of the base.



18. An octagonal prism 7 ft. high—sides of octagonal ends 3 ft. long—nearest side of prism to touch the Picture Plane 3 ft. to the left.

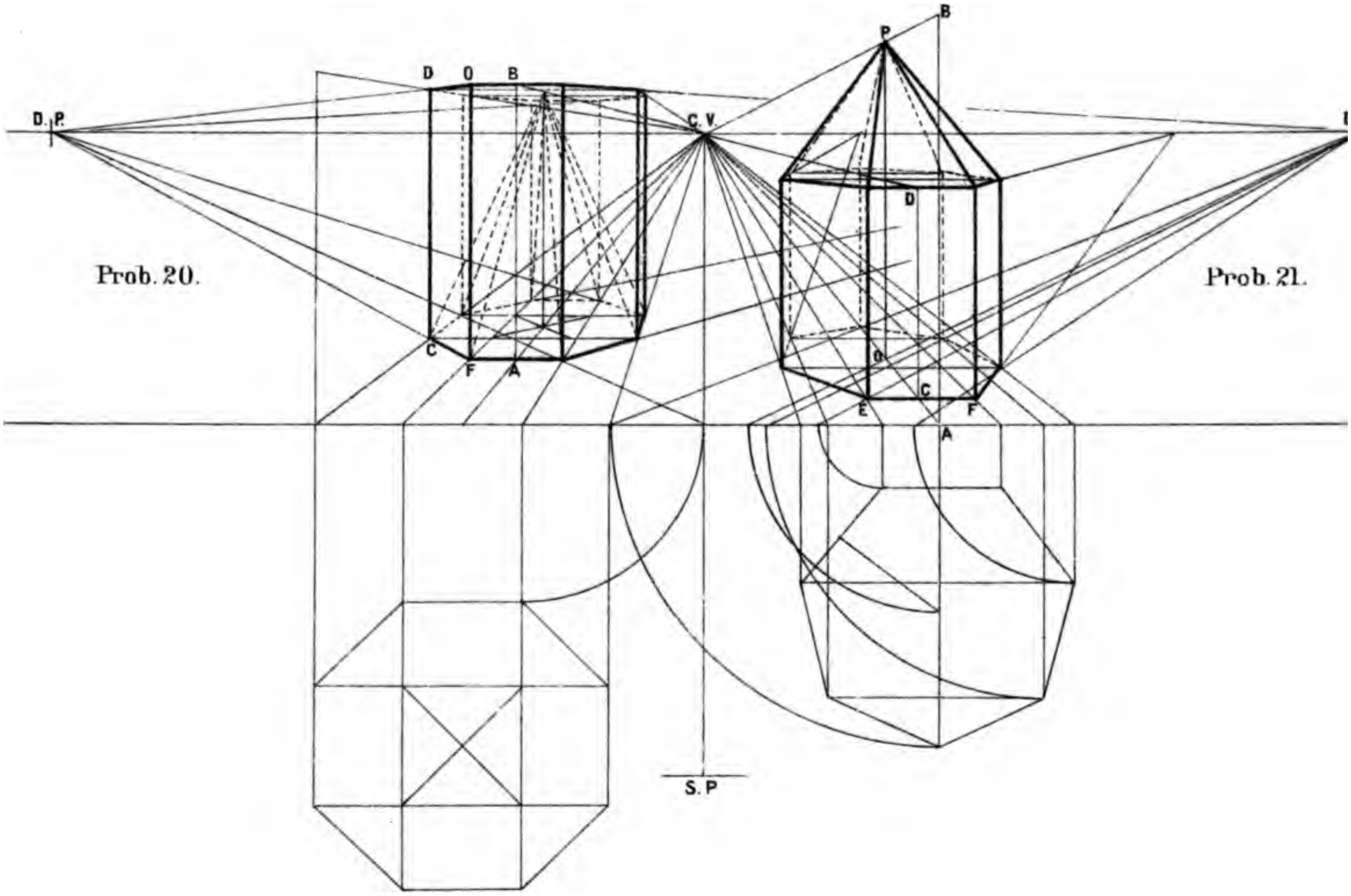
19. A pentagonal prism 7 ft. high—sides of pentagonal ends 2 ft. long—nearest side of prism to be 2 ft. to the right, 3 ft. in the picture, and parallel to the Picture Plane.

Problem 20.—The height of the prism is cut off on C D, and a line drawn through D parallel to C F (*i. e.*, from the Distance Point) gives the point O.

The apex of the pyramid, which is the centre of the octagon forming the top of the prism, is found by setting up a perpendicular on A, and drawing a line from B to the Centre of Vision. The point of intersection of this line, and the perpendicular set up on the centre of the base, is the apex.

Problem 21.—The same Height Line, A B, measures both the height of the pyramid and the height of the prism. By setting off 4 ft. on it, and drawing to the Centre of Vision, 4 ft. is cut off on the perpendicular standing on C, at the point D. A line drawn through C parallel to E F (*i. e.*, to the Picture Line) gives the front side of the top of the prism. By setting of 7 ft. on the same Height Line, and drawing to the Centre of Vision, 7 ft. is cut off on the line O P, and P is the apex of the pyramid.

The centre of the base is put in Perspective in the same way as the other points of the plan.

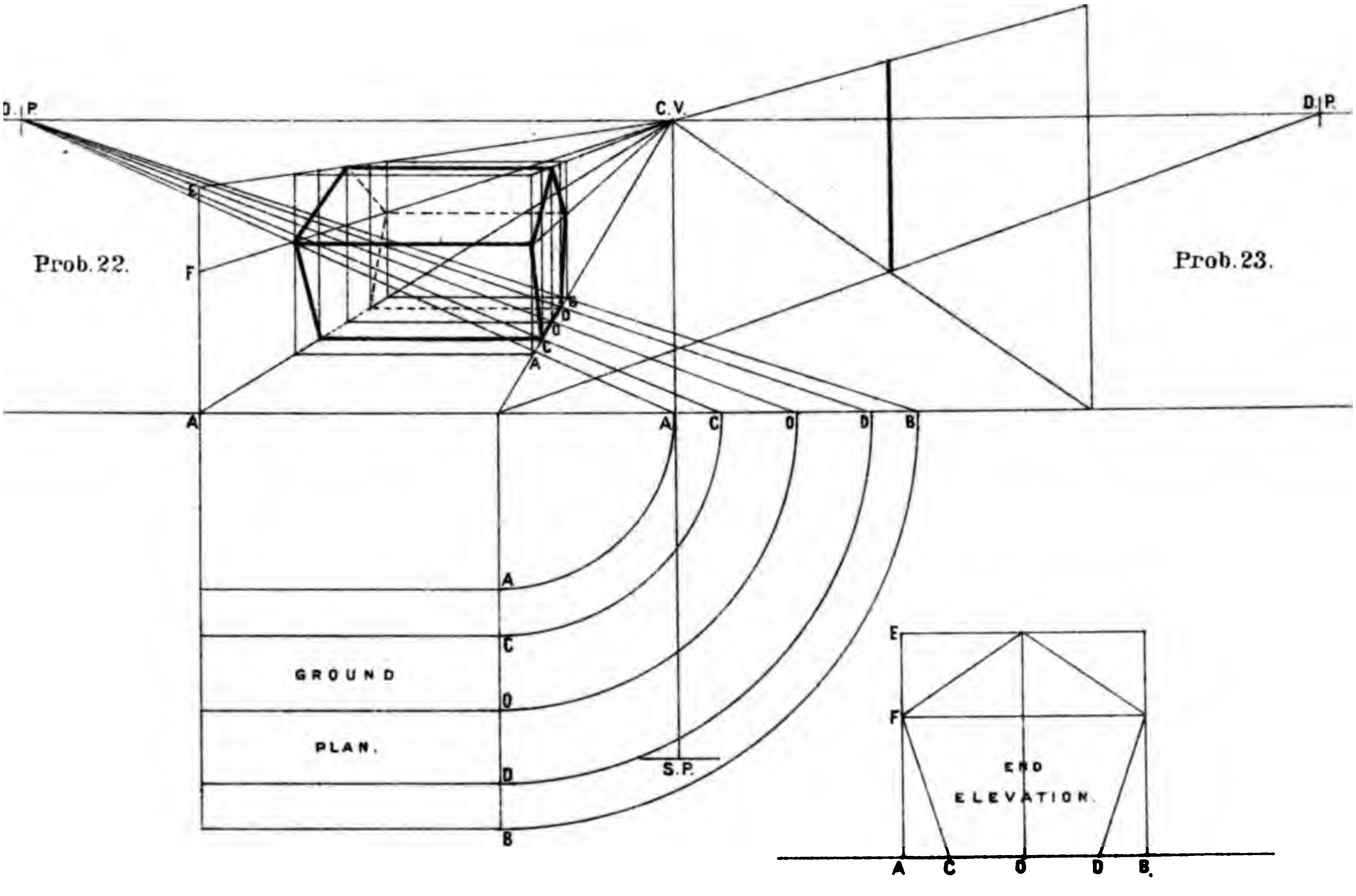


20. An octagonal prism: sides of base 2 ft. long: height of prism 6 ft. On the same base place an octagonal pyramid of the same height. Nearest side of base to be 1½ ft. to the left, 3 ft. in the picture, and parallel to Picture Plane.

21. A heptagonal prism: sides of base 2 ft. long: height of prism 4 ft. Nearest side of the base to be 1½ ft. to the right, 1 ft. in the picture, and parallel to the Picture Plane. On the top place a heptagonal pyramid 3 ft. high.

Problem 22.—Compare carefully the Perspective with the ground plan and elevation, and the working will easily be understood.

Problem 23.—This is very simple and only requires careful observation of the work.

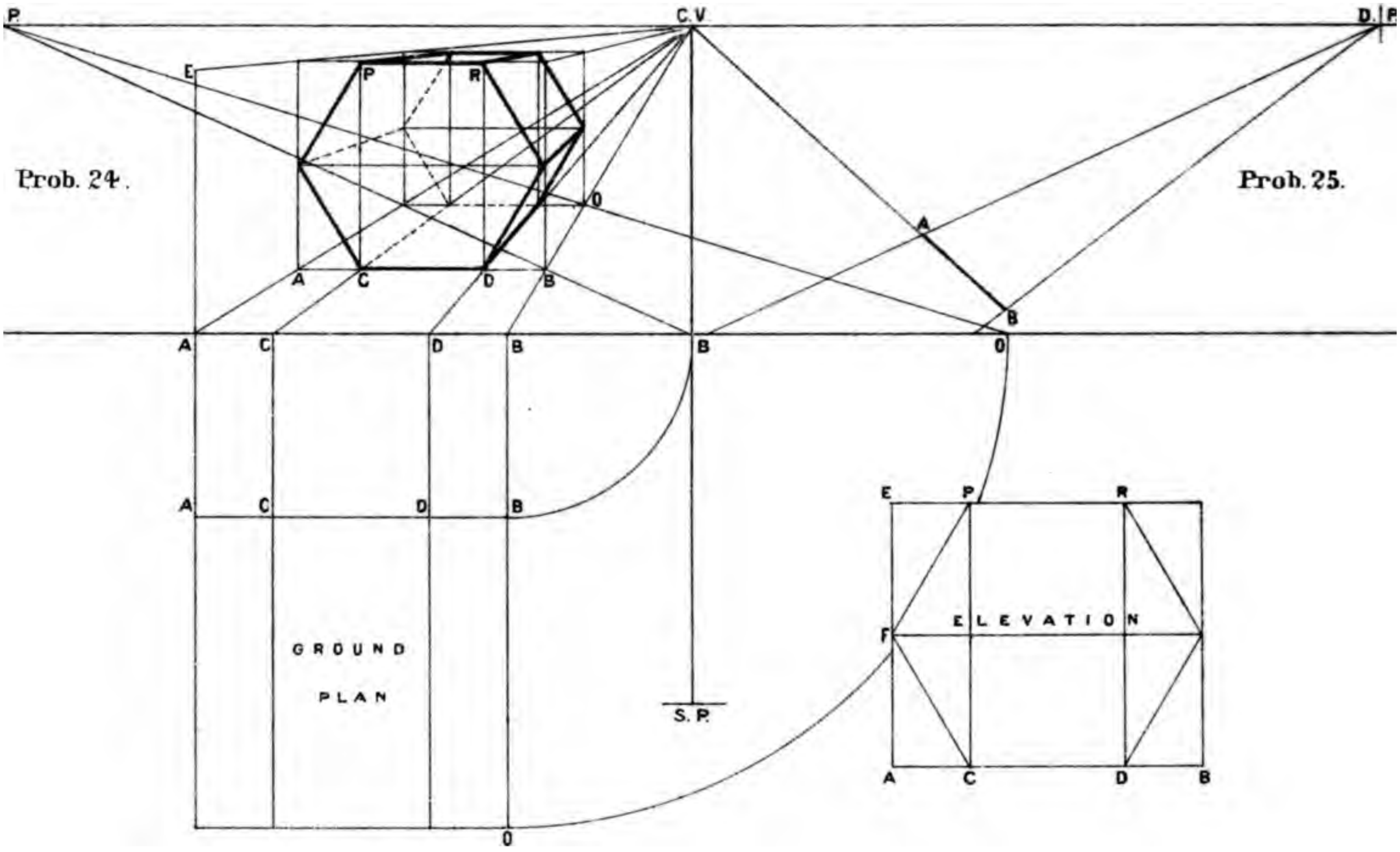


22. A pentagonal prism—sides of the end 2½ ft. long—length of the prism 5 ft.—the long edges to be parallel to the Picture Plane. Place the prism 3 ft. to the left, and 3 ft. in the picture.

23. A flagstaff 7 ft. high, 10 ft. to the right, and 10 ft. in the picture.

Problem 24.—Compare well the Perspective with the ground plan and elevation, and the working of this problem will present no difficulty.

Problem 25.—Find the Vanishing Point of A B by producing it to the Horizontal Line. All lines on the ground are measured by Measuring Points. The Measuring Point for this line will be the Distance Point, since it vanishes to the Centre of Vision. Draw then from the Distance Point through A and through B to the Picture Line—the distance between the points where these two lines meet the Picture Line is the length of A B; and this by the scale is 4½ ft.

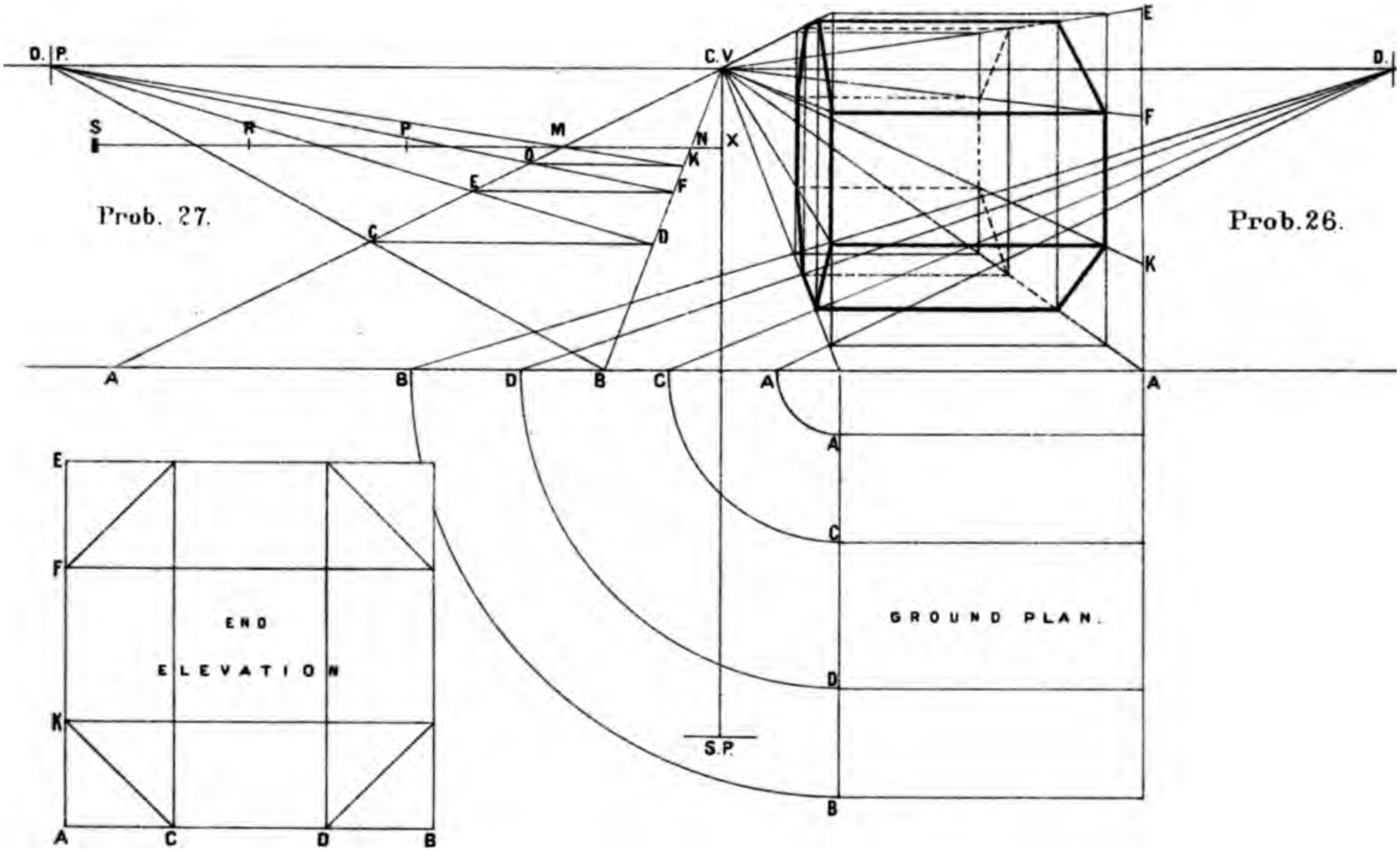


24. A hexagonal prism 5 ft. long, 3 ft. to the left, and 3 ft. in the picture: its ends being parallel to the picture plane and having 2¼ ft. sides.

25. Measure the length of the line A B given in Pespective, lying on the ground.

Problem 26.—The working of this problem will be readily understood if the ground plan and elevation be well compared with the Perspective.

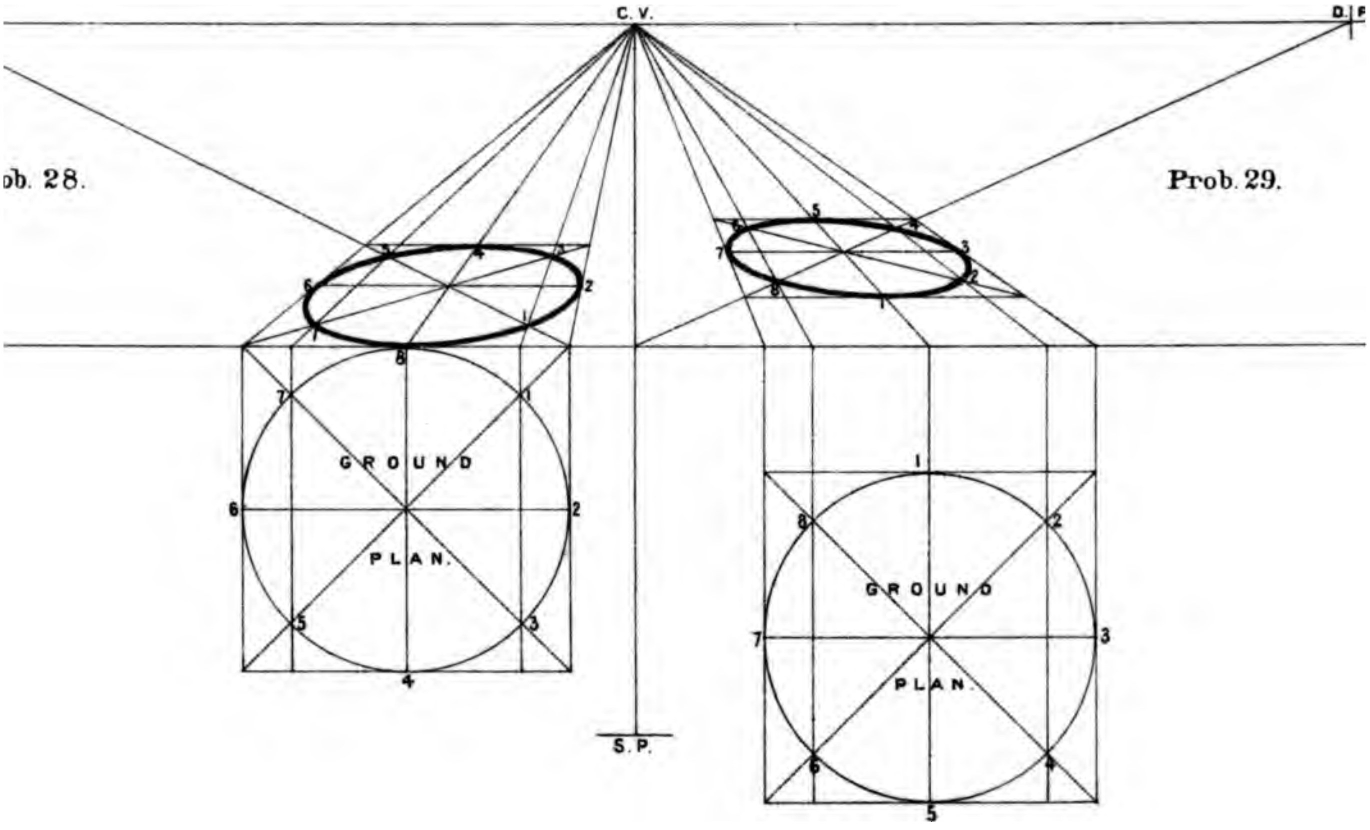
Problem 27.—To do this, find a point A on the Picture Line 10 ft. to the left and draw to the Centre of Vision. All points on the line thus drawn will be 10 ft. to the left. From A measure 8 ft. (A B) on the Picture Line, and draw from B to the Centre of Vision. Then draw from B to the Distance Point, and so find the point C 10 ft. to the left and 8 ft. in the picture. A B C D is then a square of 8 ft. Draw from D to the Distance Point, and through E draw E F parallel to the Picture-Line; E C D F is likewise a square of 8 ft., and so the point E is 10 ft. to the left and 16 ft. in the picture. In like manner O E F K is a square of 8 ft., and the point O is 10 ft. to the left and 24 ft. in the picture; and M O K N is a square of 8 ft., and the point M is 10 ft., to the left and 32 ft. in the picture. Now extend the line M N to the left, and measure on it (from M) M P, P R, and R S each equal to M X; then the point M is 10 ft. left and 32 ft. in the picture, point P is 20 ft. left and 32 ft. in the picture, point R is 30 ft. left and 32 ft. in the picture, and point S is 40 ft. left and 32 ft. in the picture.



27. Find a point 40 ft. to the left and 32 ft. in the picture, without working off the given paper.

26. An octagonal prism lying on one of its long sides (which are to be parallel to the Picture Plane) 2 ft. to the right and 1 ft. in the picture, the ends having 2½ ft. sides, and making right angles with the Picture Plane.

Problems 28 and 29.—What is said in Part I. about the Perspective of the circle being well understood and learnt, these problems will present no difficulty to the student.

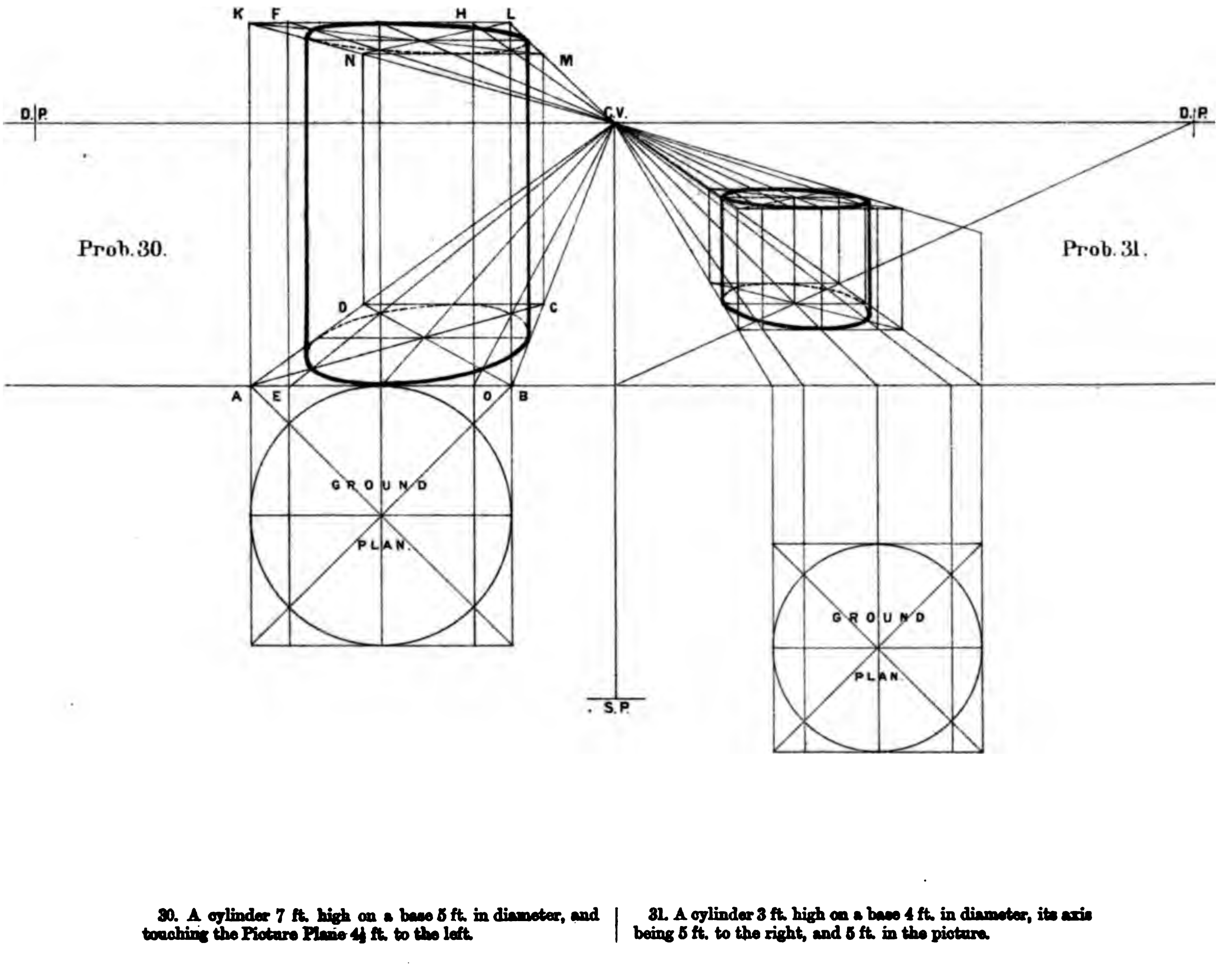


28. A circle 5 ft. in diameter touching the Picture Plane 3½ ft. to the left, lying on the ground.

29. A circle lying on the ground, 5 ft. in diameter, its centre being 4½ ft. to the right, and 4½ ft. in the picture.

Problem 30.—To obtain the circle forming the top of the cylinder, erect a solid the height of the required cylinder on the square A B C D (in which the circle forming the bottom of the cylinder is placed) as base. Then draw the diagonals and diameters of the square (K L M N) forming the top of the solid, taking care that all points are perpendicularly over the corresponding points of the bottom square. Then set up perpendiculars E F and O H and draw from F and H to the Centre of Vision. The eight points through which to draw the circle will then be obtained in the top square.

Problem 31.—The preceding problem being done and understood, this one will be easily accomplished.

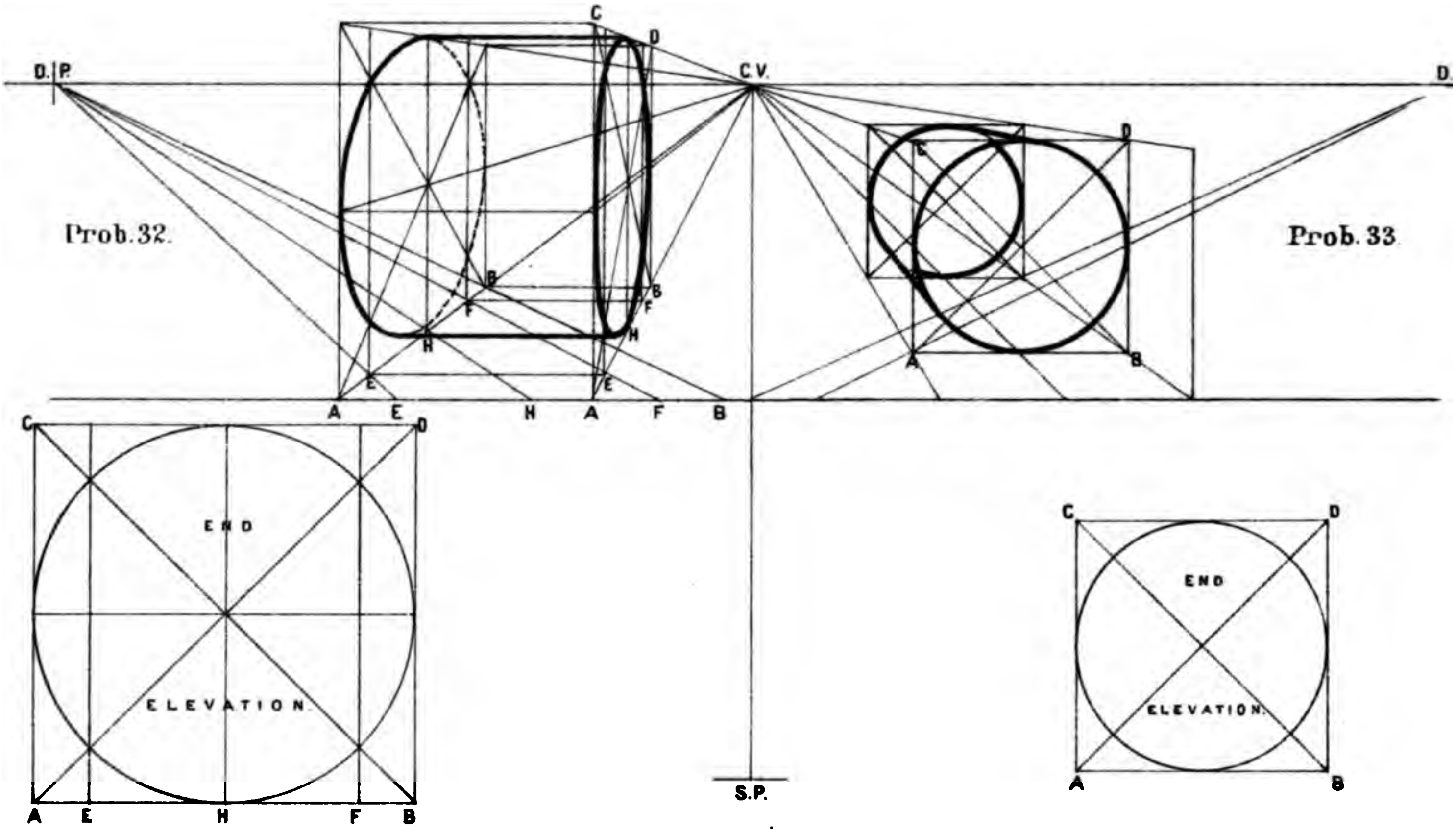


30. A cylinder 7 ft. high on a base 5 ft. in diameter, and touching the Picture Plane 4½ ft. to the left.

31. A cylinder 3 ft. high on a base 4 ft. in diameter, its axis being 5 ft. to the right, and 5 ft. in the picture.

Problem 32.—By well comparing the elevation with the Perspective, the method of working will be readily understood.

Problem 33.—The circles forming the ends of the cylinder can be drawn by the compasses, since a vertical square parallel to the Picture Plane standing in the picture is as true a geometrical square as it would be on the Picture Line.

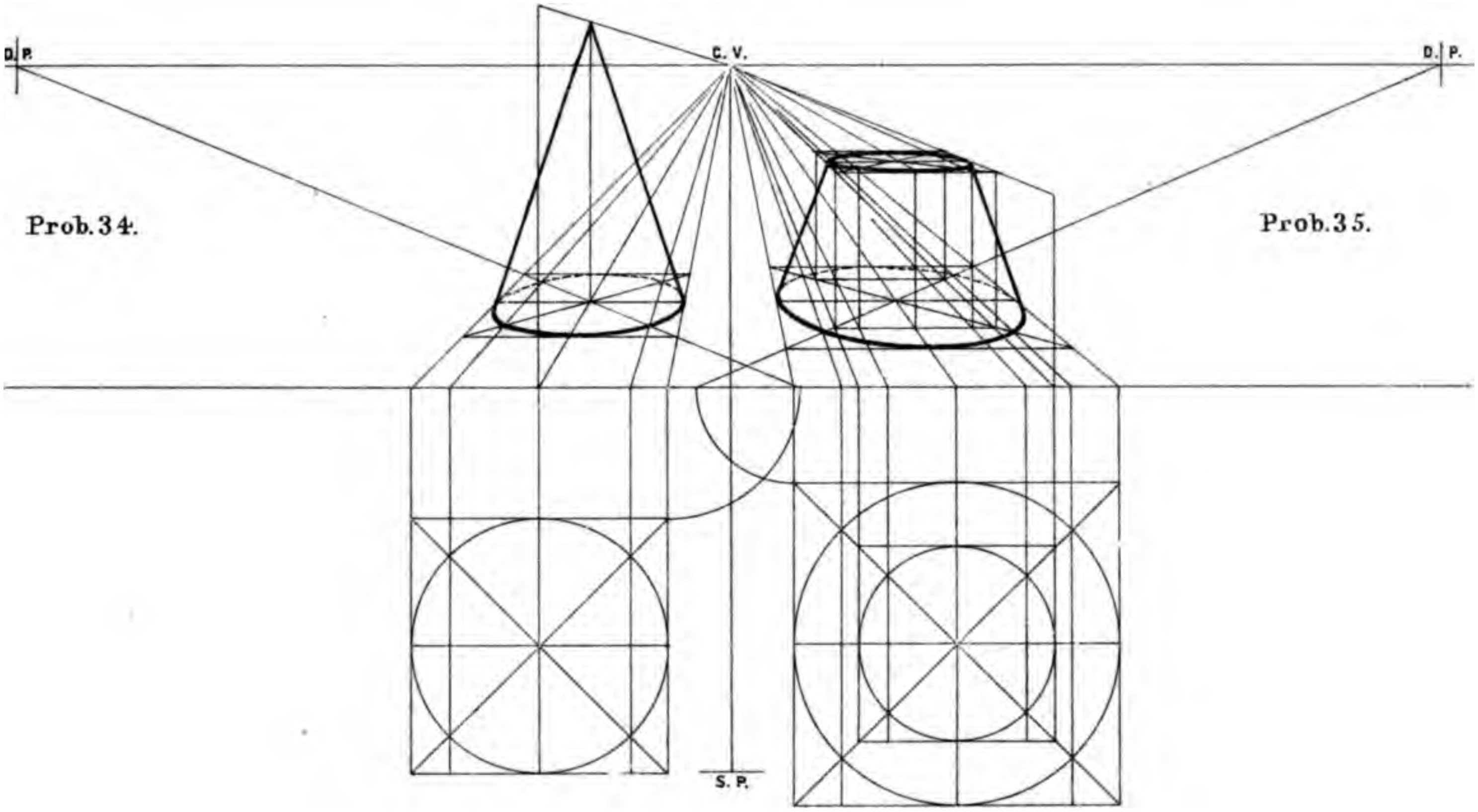


32. A cylinder 6 ft. in diameter and 4 ft. long, touching the Picture Plane; its axis being parallel to the Picture Plane and its ends making right angles with it. The nearest end to be 2½ ft. to the left.

33. A cylinder 4 ft. in diameter and 5 ft. long. Its ends being parallel to the Picture Plane, and the nearest end being 3 ft. to the right, and 2 ft. in the picture.

Problem 34.—There will be no difficulty with the working of this problem.

Problem 35.—The working of this requires care, but it is not at all difficult. To obtain the top circle put the square in which it is contained in Perspective on the ground first. Then set up a perpendicular on each corner of this square, and so obtain a like square 3 ft. above the ground, in which place the circle by the usual method.

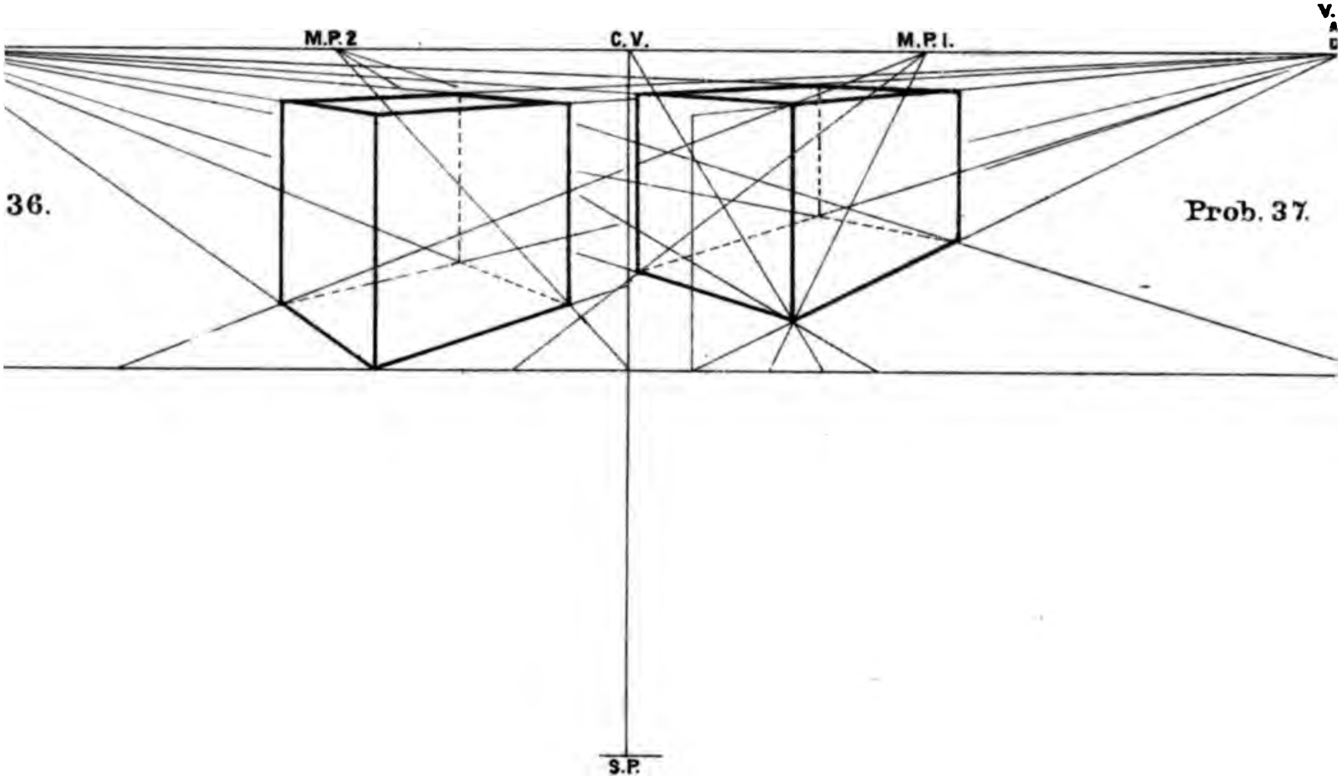


34. A cone 6 ft. high, having a base 4 ft. in diameter, and the centre of its base being 3 ft. to the left, and 4 ft. in the picture.

35. A truncated cone 3 ft. high, having base 5 ft. in diameter and top 3 ft. in diameter, and the centre of its base being 3½ ft. to the right and 4 ft. in the picture.

Problems 36 and 37.—The working of these problems is so similar to that of Problem 12 in Part I. that it will be found comparatively easy.

Observe that *distance in the picture* is found (in Prob. 37) by using the Centre of Vision and Distance Point as in Parallel Perspective—it is always found the same way.

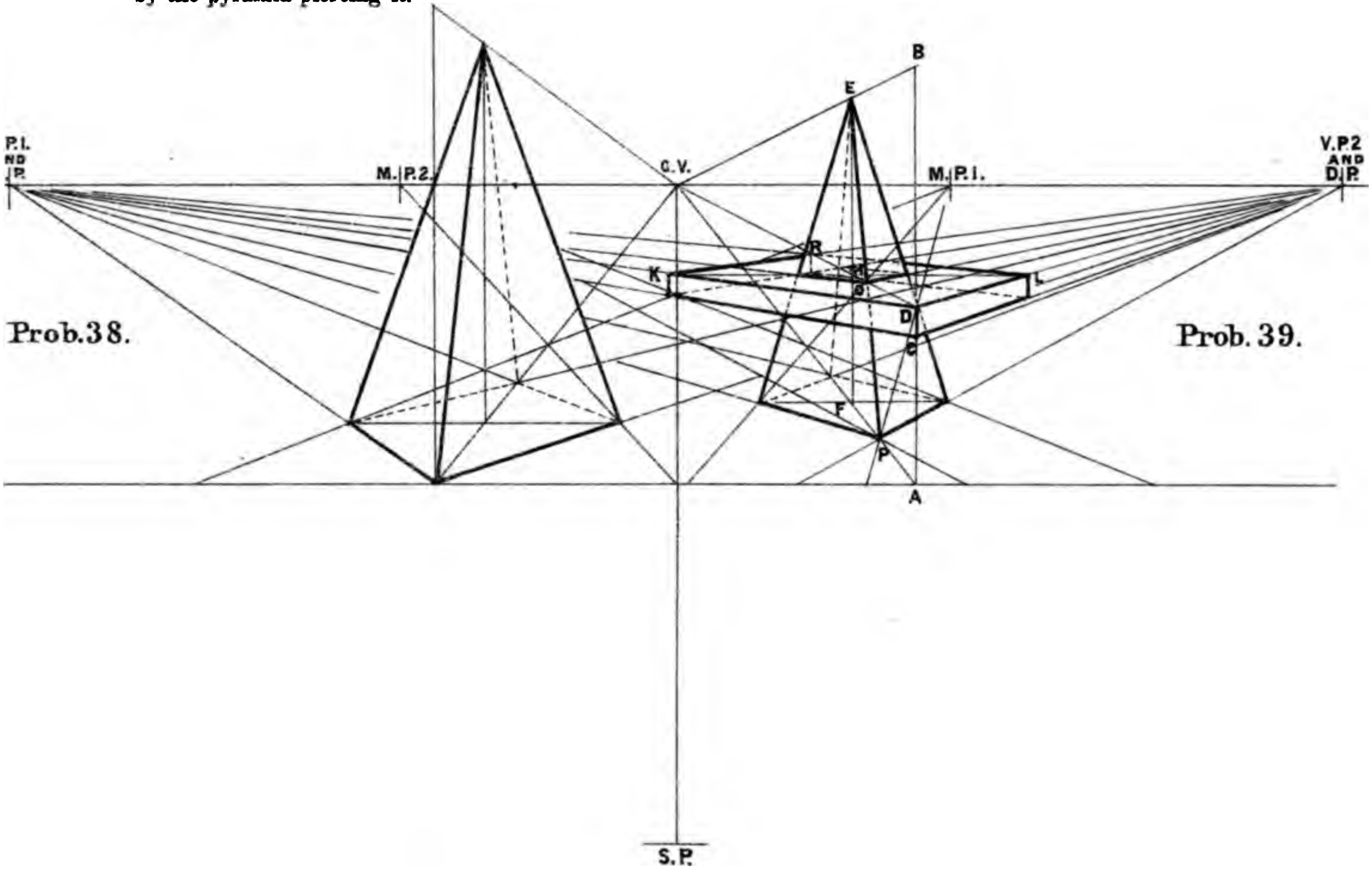


36. A cube of 4 ft., its sides making angles of 45° with the Picture Plane, and its nearest angle touching it 4 ft. to the left.

37. A rectangular solid 8 ft. long, 4 ft. wide, and 4 ft. high, lying on one of its long sides which run towards the right at angles of 45° with the Picture Plane. Its nearest angle 3 ft. to the right and 2 ft. in the picture.

Problem 38.—The working of this only requires careful observation in order to be understood.

Problem 39.—As the sides of the slab are to be parallel to the sides of the pyramid, it is clear that its angles will be exactly opposite the angles of the pyramid. The nearest angle of the pyramid is 4 ft. to the right, therefore the nearest angle of the slab will be 4 ft. to the right. And as the nearest angle of the slab is to touch the Picture Plane it will fall on the line A B, which is 4 ft. to the right and stands on the Picture Line. Measure then 2½ ft. from A to C and ½ ft. for the thickness of the slab from C to D (these will be actual scale measurements since A B stands on the Picture Line). Draw from C and D to each Distance Point. Draw from D to the Centre of Vision; this line will form one diagonal of the slab, and the point H where it cuts the perpendicular E F will be the centre of the slab. Draw through H parallel to the Picture Line and so obtain the other diagonal of the slab, which will be completed by drawing from K and L to the Distance Points. From the point O, where the diagonal (D R) of the slab cuts the angle E P of the pyramid, draw to each Distance Point to show the square hole in the slab made by the pyramid piercing it.



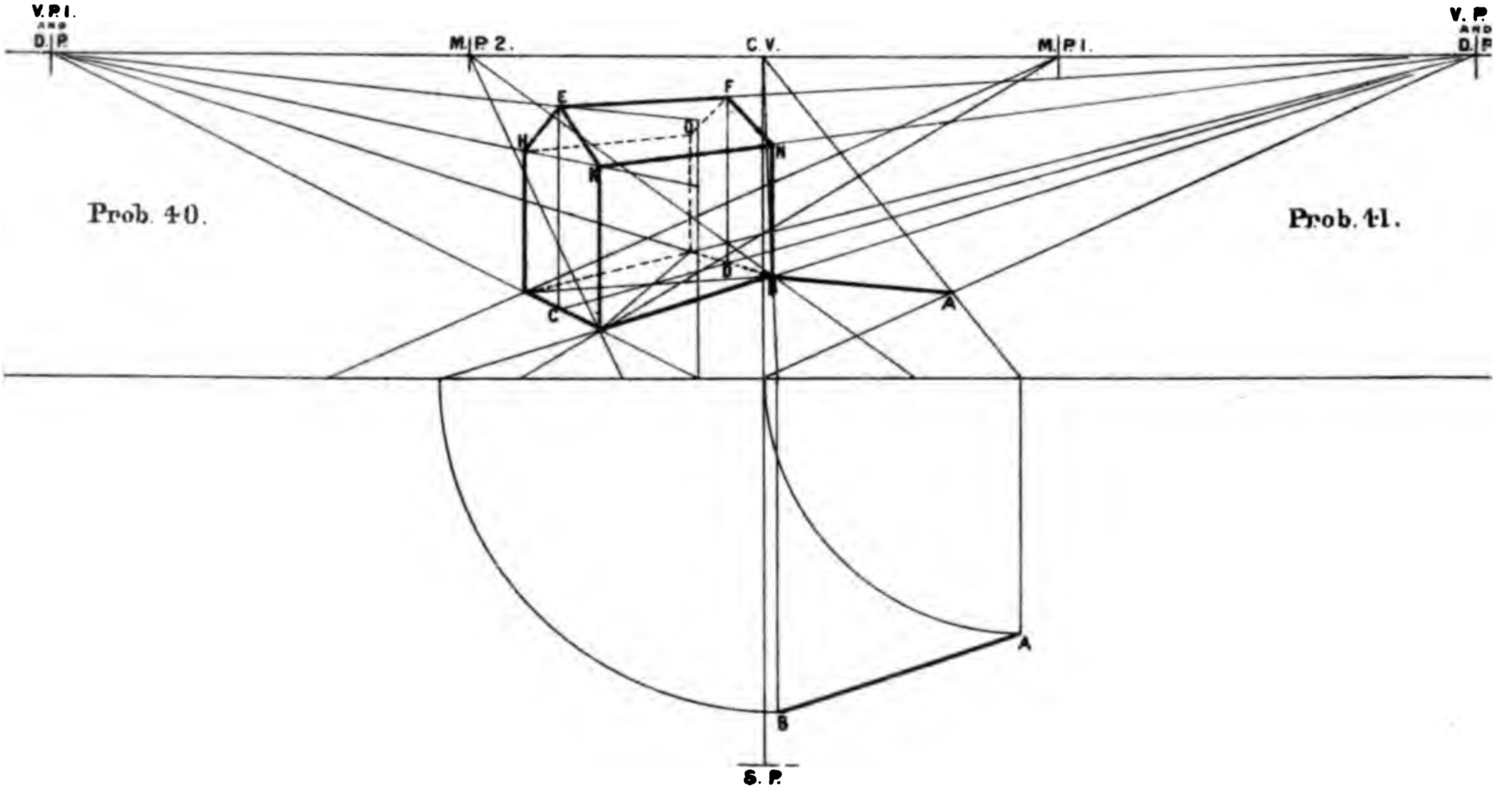
38. A pyramid 8 ft. high, on a base 4 ft. square. Nearest corner of base touching the Picture Line 4 ft. to the left. Its sides making angles of 45° with the Picture Plane.

39. A pyramid 7 ft. high on a base 3 ft. square, the nearest corner of its base being 4 ft. to the right and 2 ft. in the picture, and its sides making angles of 45° with the Picture Plane. Let a square slab be pierced through its centre by this pyramid, and rest in a horizontal position 2½ ft. from the ground, its sides being parallel to the sides of the pyramid, and its nearest angle touching the Picture Plane.

Problem 40.—To find the top line of the roof, cross the diagonals of the base and draw a line through the centre of the base, thus obtained, parallel to the long sides of the base, *i.e.*, to the Distance Point. On the points C and D, where this line cuts the short sides of the base, set up perpendiculars. Measure the 4 ft. on the one standing on C. Draw from E to the Distance Point, and the point F in which this line cuts the other perpendicular is the other end of the line forming the top of the roof. Join E H, E K and F O, F N, to complete the roof.

Problem 41.—The length of this line must be found by bringing it out of Perspective, *i. e.*, by making a plan of it; and this is done by finding each of the points A and B in plan and joining them. The working will be at once apparent; for, knowing how to put a point *into* Perspective from plan, it will be very easy to see how each of these points is brought *out of* Perspective.

If the Vanishing Point of the line could be conveniently found, it would be neater to find its length by means of the Measuring Point.

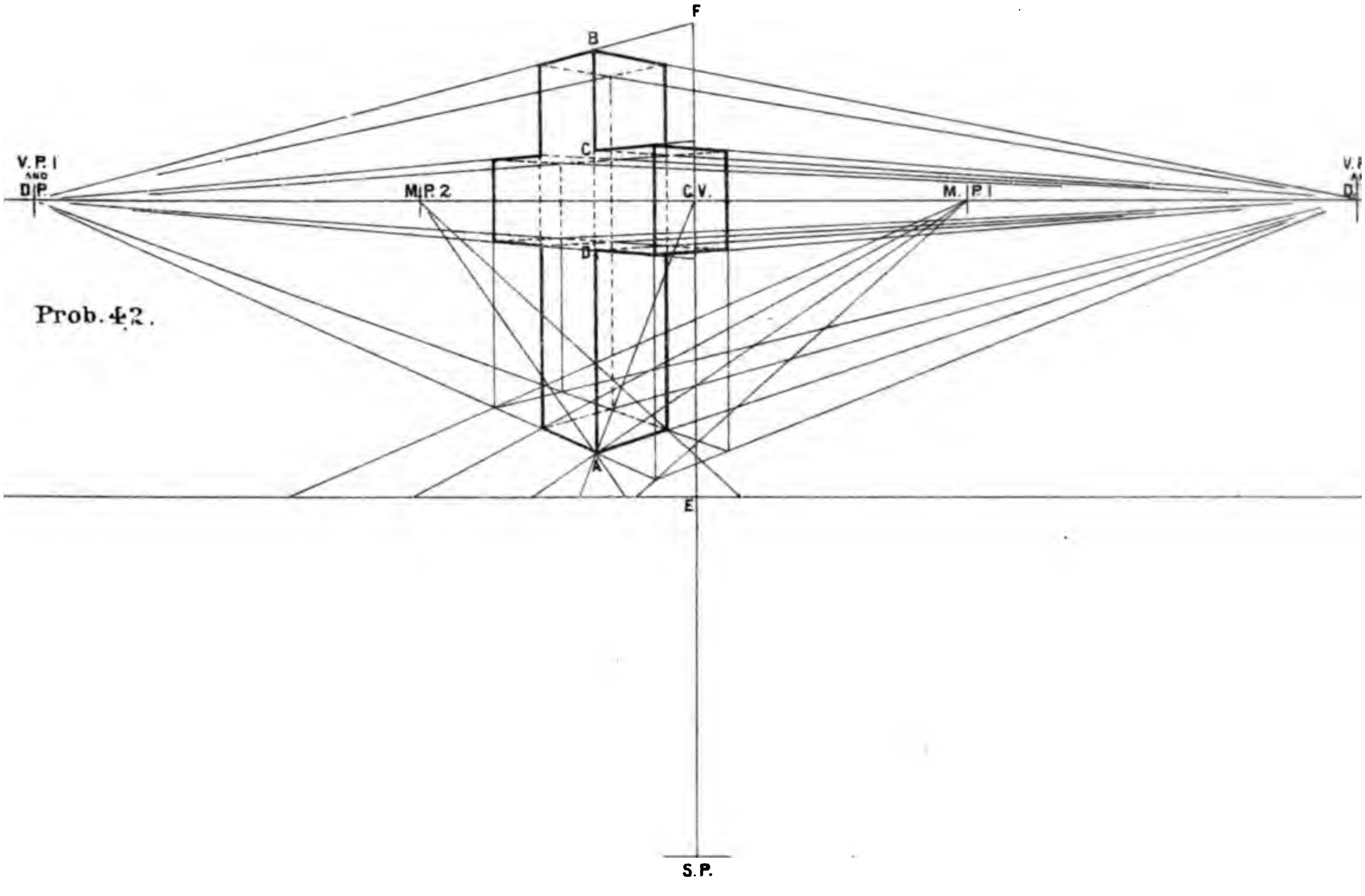


40. A house-shaped solid at angles of 45° with the Picture Plane. The nearest corner 3 ft. to the right and 2 ft. in the picture. Its base or floor, 4½ ft. long and 3 ft. wide. Its sides or walls 3 ft. high. Gable of the roof 4 ft. high. Long sides to run to the right.

41. Find a point 4 ft. to the right and 4 ft. in the picture, and mark it A. Draw a line from this point to the angle of the base of the solid in the preceding problem marked B; then find the length of the line A B.

Problem 42.—The 8 ft. for the height, 2 ft. from the top, and 2 ft. for the width of the arms are all measured on the line A B by means of the height line E F; and through the points C and D from Vanishing Point 1 are drawn the front edges of the arms.

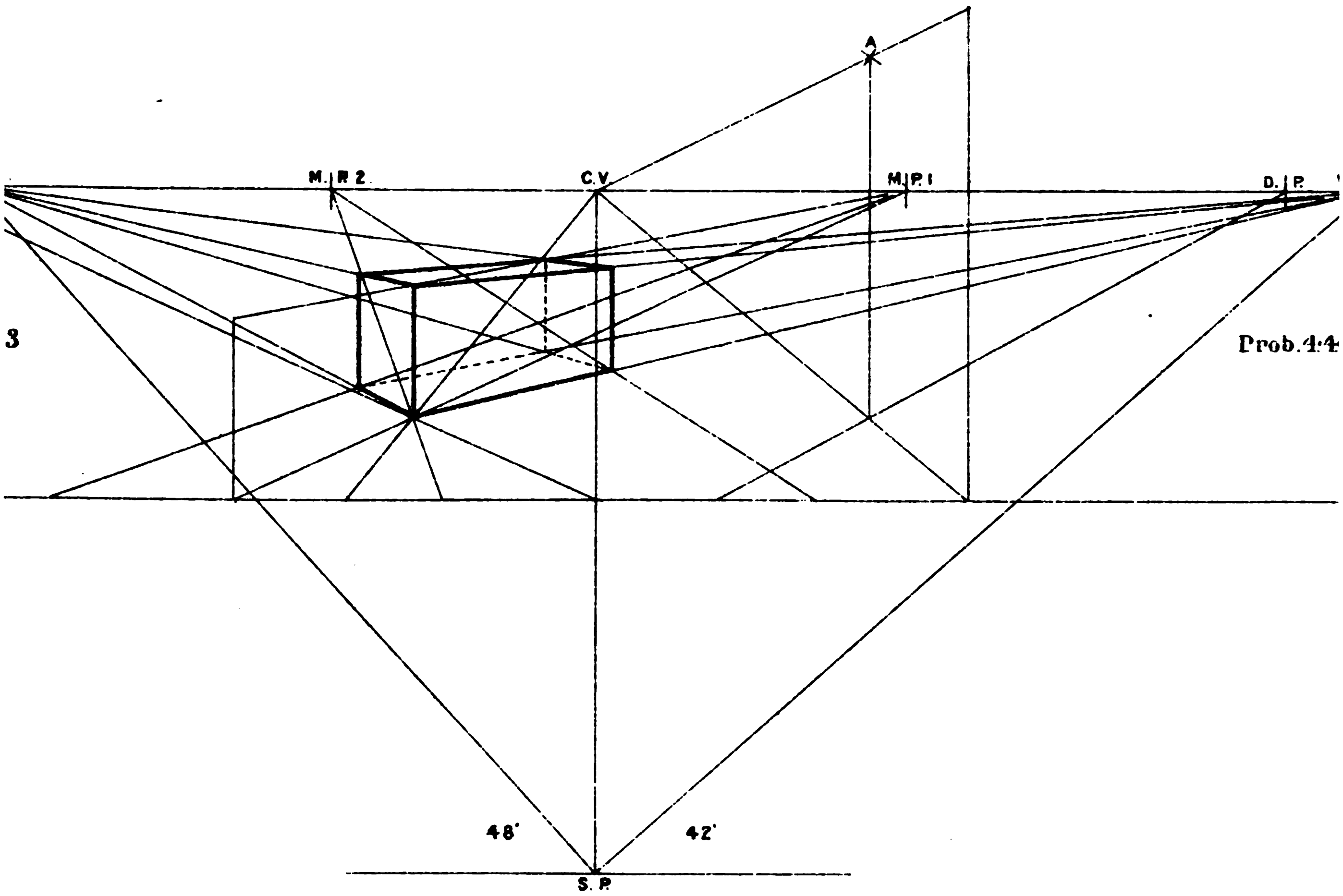
The corners of the arms are obtained by perpendiculars carried up from work previously done on the ground.



42. A solid cross 8 ft. high. The ends of the shaft and of the arms 2 ft. square. The arms 2 ft. long, and 2 ft. from the top of the shaft, and running lengthwise towards the left.

Problem 43.—If the sides of the solid make angles of 42° with the Picture Plane towards the right, it is plain that the ends must make angles of 48° with it towards the left, since the angle included by the side and end of the solid is a right angle.

Problem 44.—If the working of this be carefully observed it will be easily understood, since similar things have already been done in the working of preceding problems.



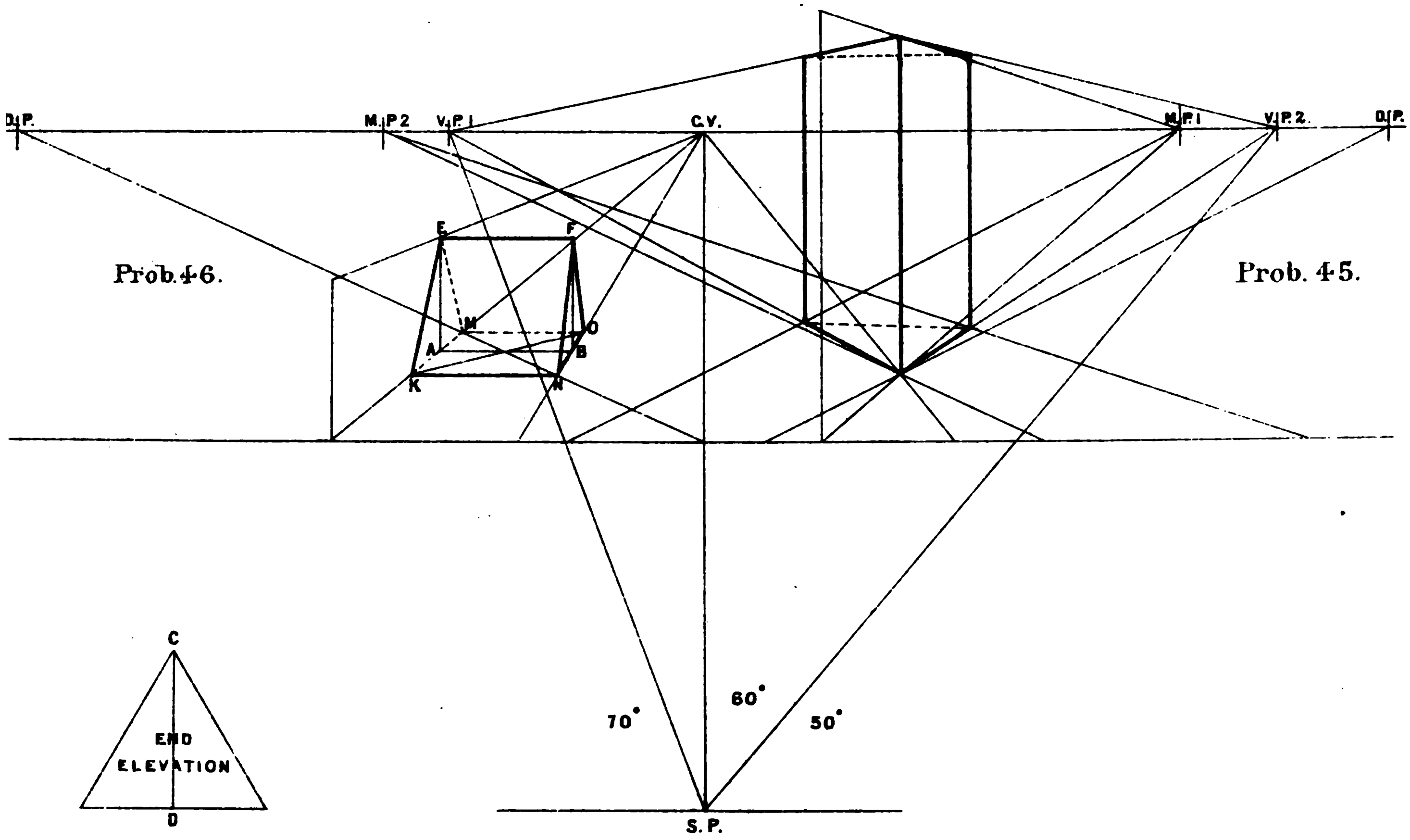
43. A rectangular solid 6 ft. long, having ends 3 ft. square, lying on the ground, with its long edges running towards the right at an angle of 42° with the Picture Plane. The nearest corner to be 4 ft. to the left and 4 ft. in the picture.

44. Find, and mark A, a point 6 ft. to the right, 4 ft. in the picture, and 8 ft. from the ground.

Problem 45.—The angle of an equilateral triangle is 60°; therefore if one side makes an angle of 50° towards the right, the other must make an angle of 70° towards the left, since the three angles must make up two right angles, *i.e.*, 180°.

Two sides of the triangle being obtained, the third side is got by joining the extremities of these.

Problem 46.—The prism being 3 ft. long, and its ends having 3 ft. sides, the side on which it lies is a square of 3 ft. To obtain the ends, cross the diagonals of the side on which the prism lies, and through the centre of this side, thus obtained, draw a line (A B) parallel to the Picture Line. On A and B set up perpendiculars and measure off on each the altitude of the triangle (*i.e.*, C D, in the End Elevation); then join E K, E M and F N, F O. To complete the prism, join E F.

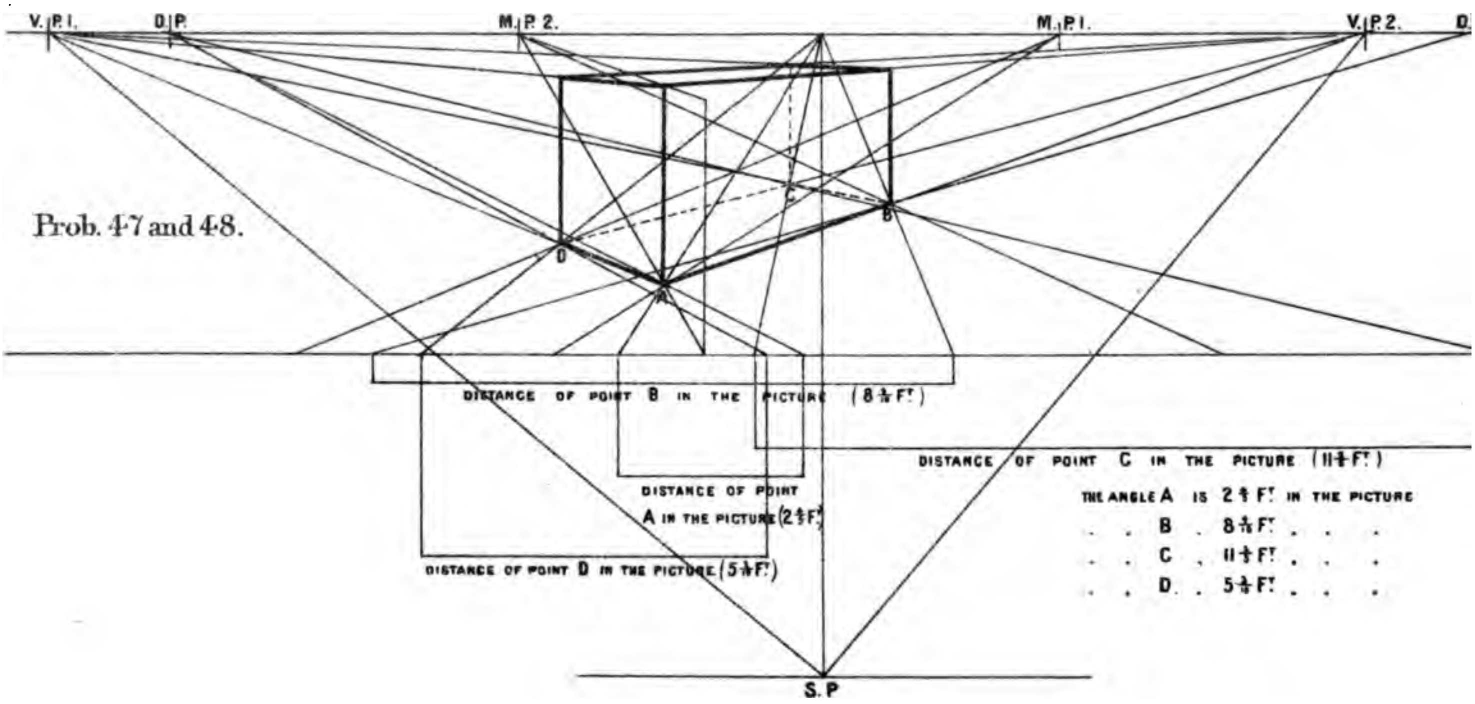


46. An equilateral triangular prism 3 ft. long, lying on one of its sides, which are parallel to the Picture Plane, 3 ft. to the left and 3 ft. in the picture. The sides of the triangular ends are 3 ft. long.

45. A prism 7 ft. high, standing on an equilateral triangular base of 3 ft. sides. One side of the base to recede towards the right at an angle of 50° with the Picture Plane. The nearest angle to be 4 ft. to the right and 3 ft. in the picture.

Problem 47.—This being very like Problems 37 and 43 will not be found difficult.

Problem 48.—Draw through each of the points A, B, C, D, a line from the Centre of Vision, and likewise one from the Distance Point to the Picture Line. The distance between the points where these two lines meet the Picture Line is the distance of the angle within the picture.


V.P.1.
D.P.
M.P.2.
M.P.1.
V.P.2.
D.
A
B
C
D
DISTANCE OF POINT B IN THE PICTURE (8 3/16 FT.)
DISTANCE OF POINT C IN THE PICTURE (11 1/5 FT.)
DISTANCE OF POINT A IN THE PICTURE (2 4/5 FT.)
DISTANCE OF POINT D IN THE PICTURE (5 3/16 FT.)
THE ANGLE A IS 2 4/5 FT. IN THE PICTURE
" " B " 8 3/16 FT. " " "
" " C " 11 1/5 FT. " " "
" " D " 5 3/16 FT. " " "
S.P

Prob. 47 and 48.

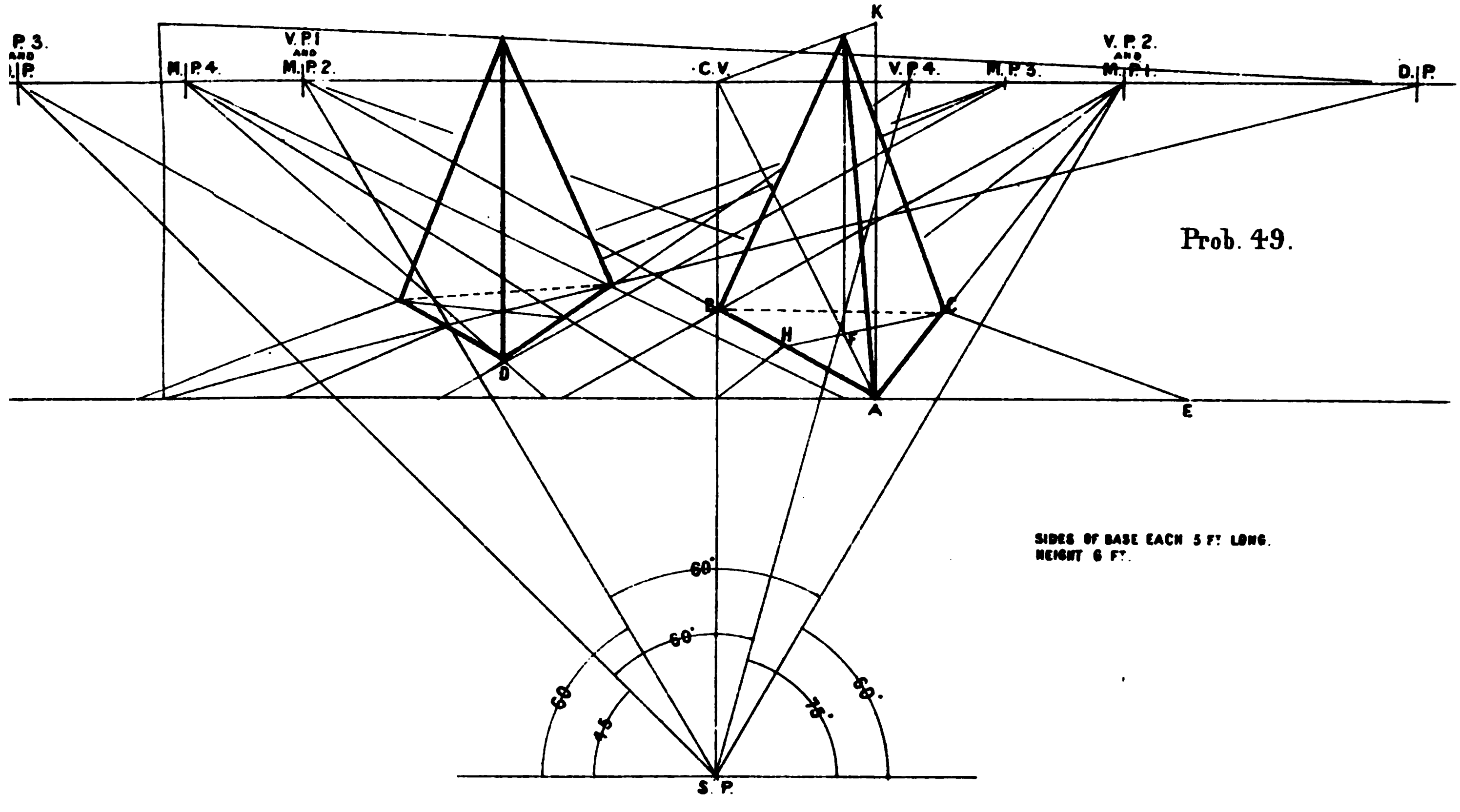
47. Take any point A between the ground line and horizon, and on the left of the Line of Direction. At this point is the nearest angle of a rectangular solid whose end is 4 ft. square, and length 8 ft. The solid lies upon one of its long sides which incline towards the right hand at an angle of 50° with the Picture Plane. Draw the Perspective representation of the solid.

48. Find how far each angle of the foregoing figure is from the Picture Plane.

Note.—The Distance, or Line of Direction = 10 ft.

Problem 49.—*(This is a Second Grade Examination Paper.)* Find the Vanishing Points of A B and A C by producing them to the Horizontal Line. Draw from these Vanishing Points to the Station Point, and the included angle will be found to be 60°; this shows that the base of the pyramid is an equilateral triangle, and therefore if one side of it be measured that will be sufficient. The Vanishing Point of one line is, in this case, the Measuring Point of the other, as will be seen by following the rule for finding Measuring Points, and A C will be measured by drawing from Measuring Point 2 (*i.e.*, Vanishing Point 1) through C to the Picture Line. The distance from A to E on the Picture line, is the length of A C in Perspective, and this is 5 ft. The base then is an equilateral triangle of 5 ft. sides. Next measure the height. This, of course, must be measured from the centre of the base to the apex. Find then the centre of the base. This can be done by bisecting two of its sides and drawing from the points of bisection to the opposite angles. A C will be bisected by drawing from A to the Centre of Vision, for B C is parallel to the Picture Line and the point A is exactly opposite its centre, since A B C is an equilateral triangle. Measure 2½ ft. from A, along the picture to the left, and draw from the point thus found to Measuring Point 1; this line bisects the side A B in the point H. Draw from H to C and the point F is the centre of the base. Draw a perpendicular from F to the apex; measure this by the Height Line A K, and it will be found to be 6 ft.

In placing the same figure on the left, first find the Vanishing Points. One is to be that for lines making angles of 45° with the Picture Plane, therefore, since the included angle must be 60°, the other will be the Vanishing Point for lines making angles of 75° with the Picture Plane.

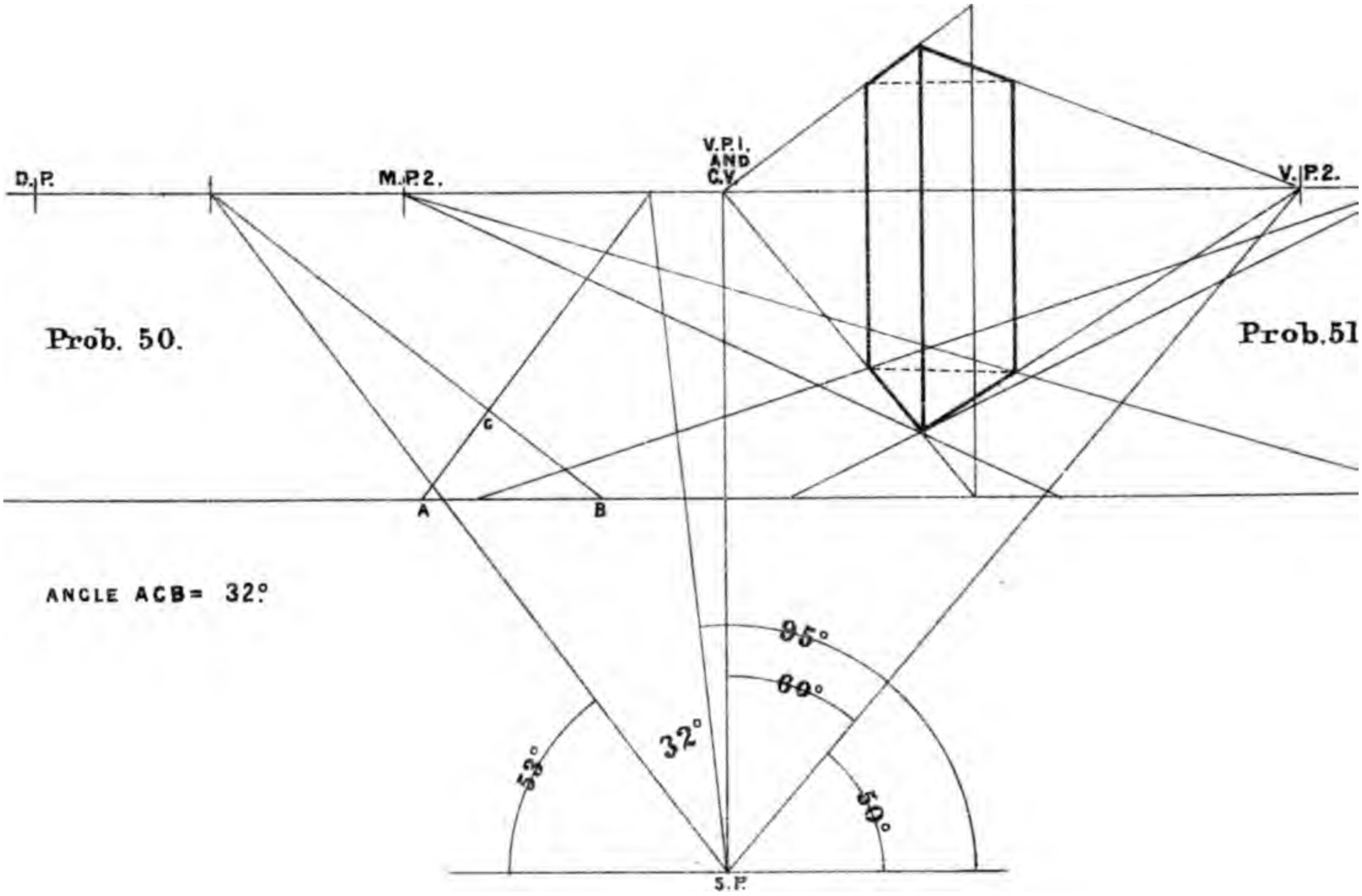


Prob. 49.

49. Find the dimensions of the solid given in Perspective on the right, and put it in Perspective again on the left, so that the point A shall be on D, and the line A B retire to the left at an angle of 45° with the Picture Plane.

Problem 50.—Produce A C and B C to the Horizontal Line, and so find their Vanishing Points. Draw lines from these Vanishing Points to the Station Point. This will show the angle which each of the lines A C and B C makes with the Picture Plane, and it will be seen that A C makes an angle of 95° to the right, and B C one of 53° to the left. Now the three angles of a triangle always together equal two right angles (*i.e.*, 180°). If then the angle at A = 95°, and that at B = 53°, it is clear that the angle at C must equal the remainder of the 180°, *i.e.*, 180°—(95° + 53°) = 32°.

Problem 51.—If one side of the triangular base is to make an angle of 50° towards the right, and the included angle is to be 40°, it is plain that the side which runs towards the left must make an angle of 90° with the Picture Plane, for 50° + 40° + 90° = 180°.

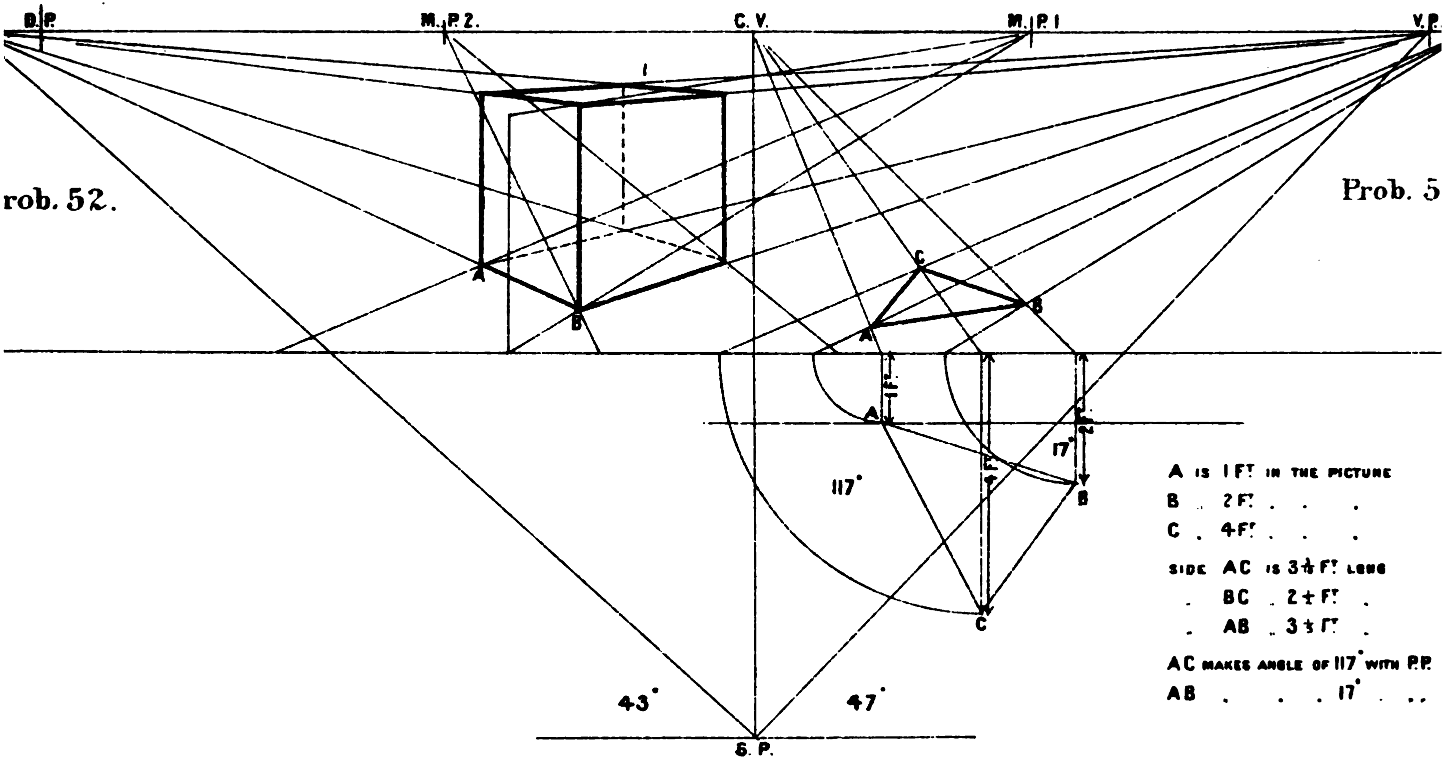


50. Upon the Ground Line on the left of the spectator take two points A and B. Let A B be the front side of a triangle upon the ground; and take any point C beyond the Ground Line to represent the vertex of the triangle. Show the shortest method of measuring the angle A C B.

51 A. prism 8 ft. high standing on a triangular base which has one side 6 ft. long, and another 5 ft. long, and these two sides including an angle of 40°. The 7 ft. side of the base to run to the right at an angle of 50°, and the 6 ft. side to run to the left. Nearest angle to be 4 ft. to the right, and 3 ft. in the picture.

Problem 52.—First find the Vanishing Point of A B by producing it to the Horizontal Line. Then draw a line from this Vanishing Point to the Station Point, and so find the angle which A B makes with the Picture Plane. Add 90°(the angle of the base of the cube) to this and subtract the sum from 180°; the remainder will be the angle which will be made by the side of the base running towards the right, for which find the Vanishing Point—of course each edge of the cube must be the length of A B, and this is found on the Picture Line, by drawing lines from Measuring Point 1 through A and B to the Picture Line.

Problem 53.—Make a Ground Plan of the triangle by bringing the three points A, B, C, out of Perspective (according to the method followed in Problem 41) and joining them. From this plan may be measured all that is required,—viz., distances in the picture, length of sides, and angles made with the Picture Plane.

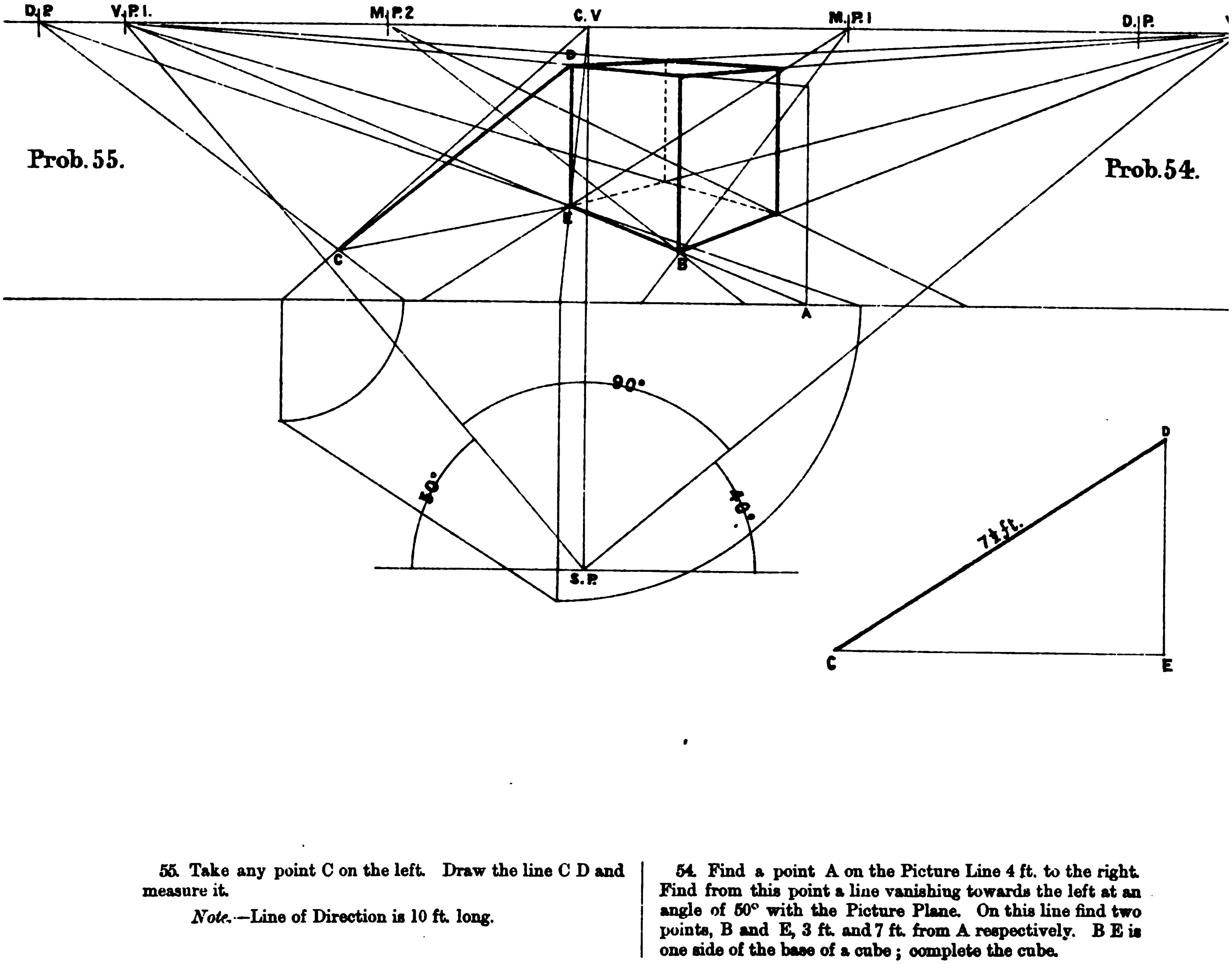


52. Upon the left take any two points A and B between the Ground Line and Horizontal Line, and join them. Let A B represent one side of the base of a cube. Complete the cube.

53. Take any points A, B, C, not in the same straight line between the Ground Line and horizon, and on the right of the spectator. Find how far each point is beyond the Ground Line, and joining the points to form a triangle, measure each side and the angles made with the Picture Plane.

Problem 54.—This being similar to Problem 52 will be easily accomplished.

Problem 55.—As the point C is on the ground, and point D is 4 ft. above it, it is clear that the line C D is neither parallel nor perpendicular to the ground, but is oblique to it, making an angle with it smaller than a right angle. Line C E is on the ground. and D E is perpendicular to the ground, therefore the angle C E D is a right angle, and the triangle D C E is a right-angled triangle, standing vertically on the ground, on C E as base. If then we find the length of C E, and place D E (which we know is 4 ft. long) at right angles to it, at its extremity E, and join C D, we shall have a geometrical representation of the triangle, and C D can be measured from it with the compasses and scale. Bring the line C E out of Perspective in the usual way. Make C E, thus brought out, the base of the triangle, and place D E at right angles to it, at the point E; then join D C. D C is 7½ ft. long.



55. Take any point C on the left. Draw the line C D and measure it.

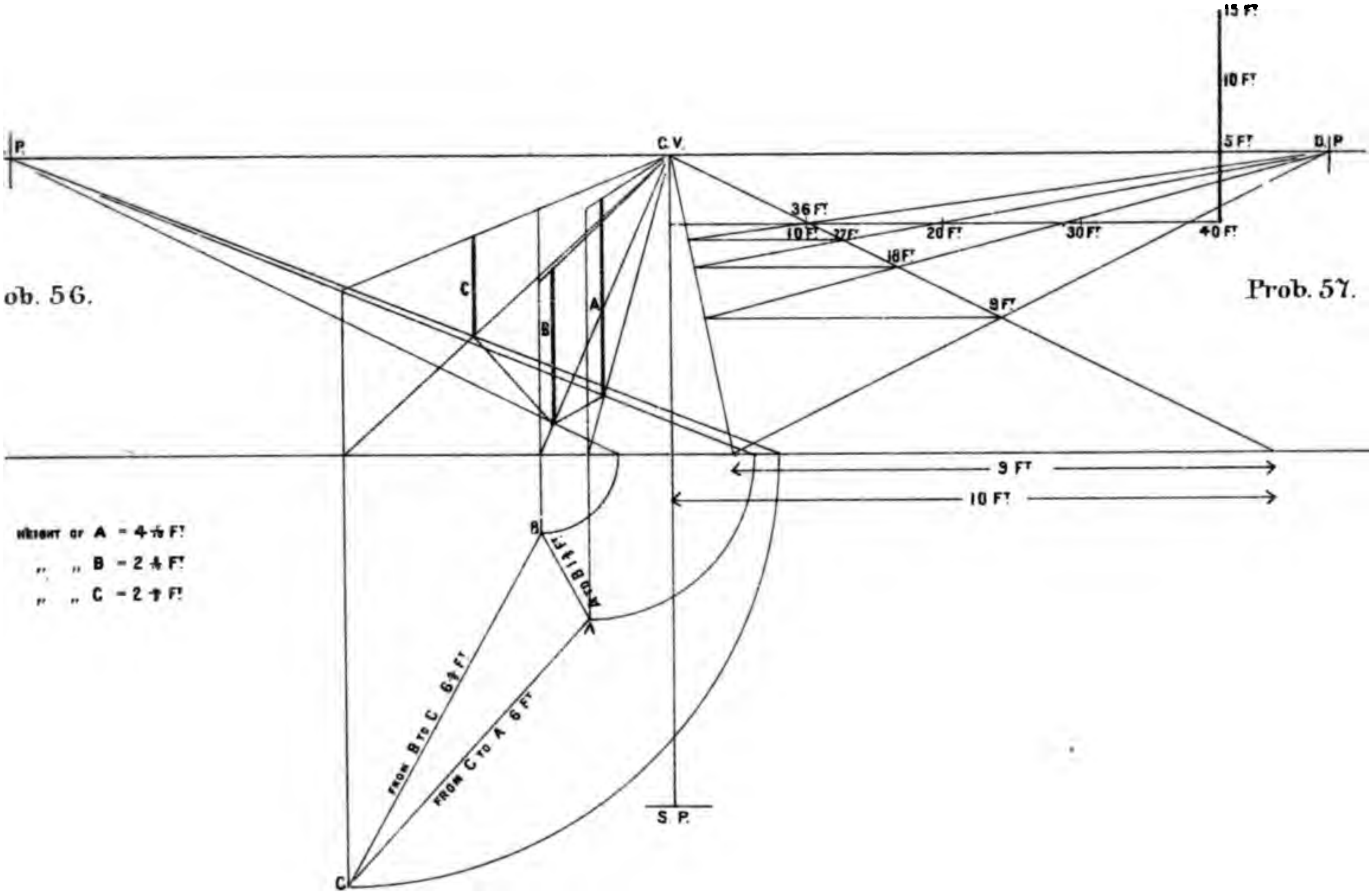
Note.—Line of Direction is 10 ft. long.

54. Find a point A on the Picture Line 4 ft. to the right. Find from this point a line vanishing towards the left at an angle of 50° with the Picture Plane. On this line find two points, B and E, 3 ft. and 7 ft. from A respectively. B E is one side of the base of a cube; complete the cube.

Problem 56.—Join the lower extremities of the lines to form a triangle. Bring this triangle out of Perspective. The sides of this plan triangle will be the actual distances of the lines from each other.

The length of the sides of the triangle could likewise be obtained by finding the Vanishing Point of each line and then measuring it by the Measuring Point; but this method would necessitate three Vanishing Points and their Measuring Points, and lines running to each of these six points would be likely to confuse. However, the student can try both methods.

Problem 57.—The working of this problem is much like that of Problem 27. The flagstaff stands on the ground. From the ground to the Horizontal Line is 5 ft. Therefore three times the distance from the foot of the flagstaff to the Horizontal Line, will give the required height,—viz., 15 ft.



ob. 56.

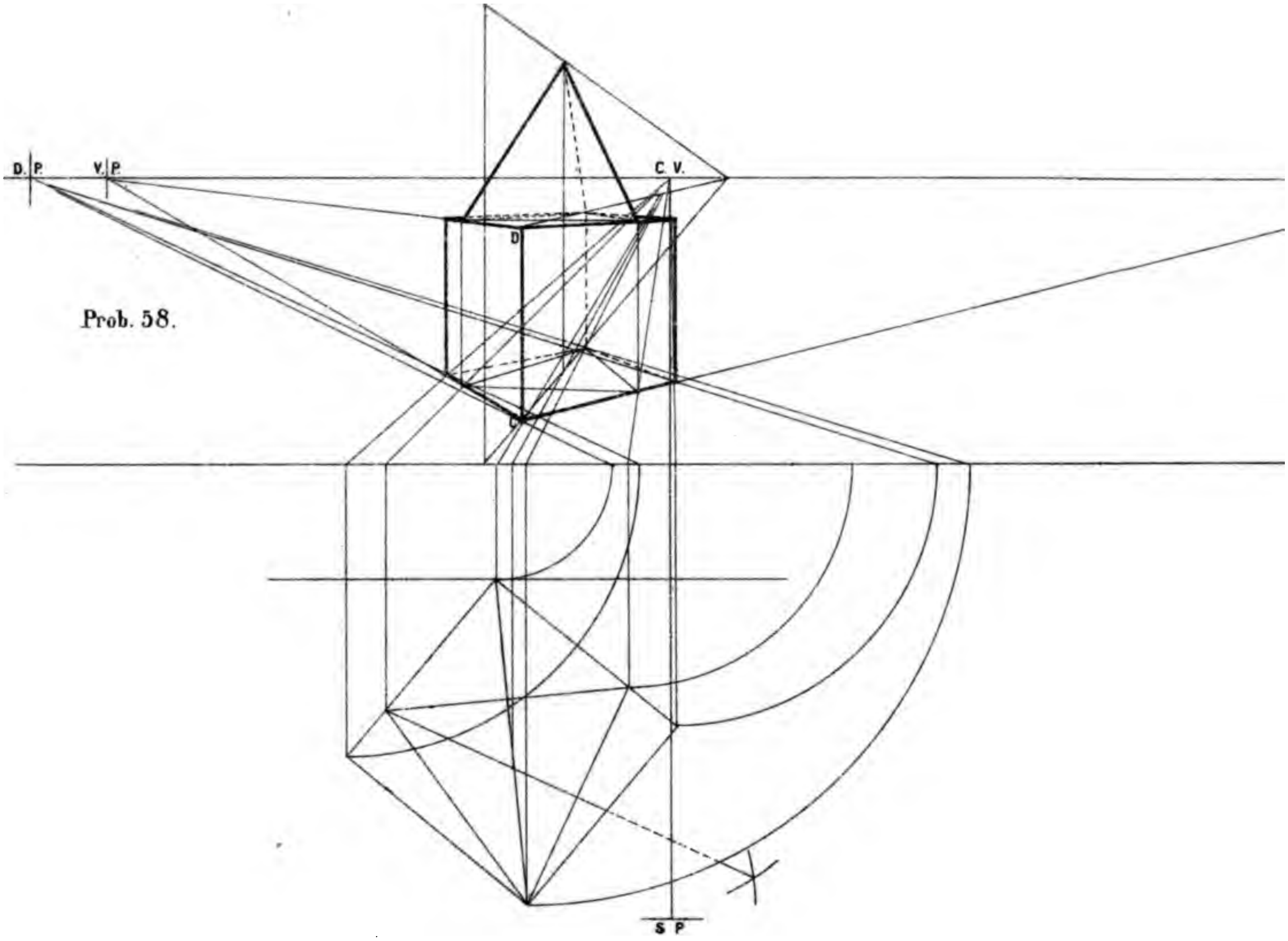
Prob. 57.

56. A, B, and C are three vertical lines standing on the ground. Find the height of each, and the distances from each other.

57. Erect a flagstaff 15 ft. high, 40 ft. to the right, and 36 ft. in the picture.

Problem 58.—It is best to work this problem from a Ground Plan. The centre of the base of the pyramid falls on the diagonal of the square, so that the Height Line, set up where the diagonal of the square produced meets the Picture Line, is used to measure both the height of the pyramid, and the height of the cube on the nearest edge C D.

The centre of the triangle is put into Perspective in the same way as the other points of the plan are



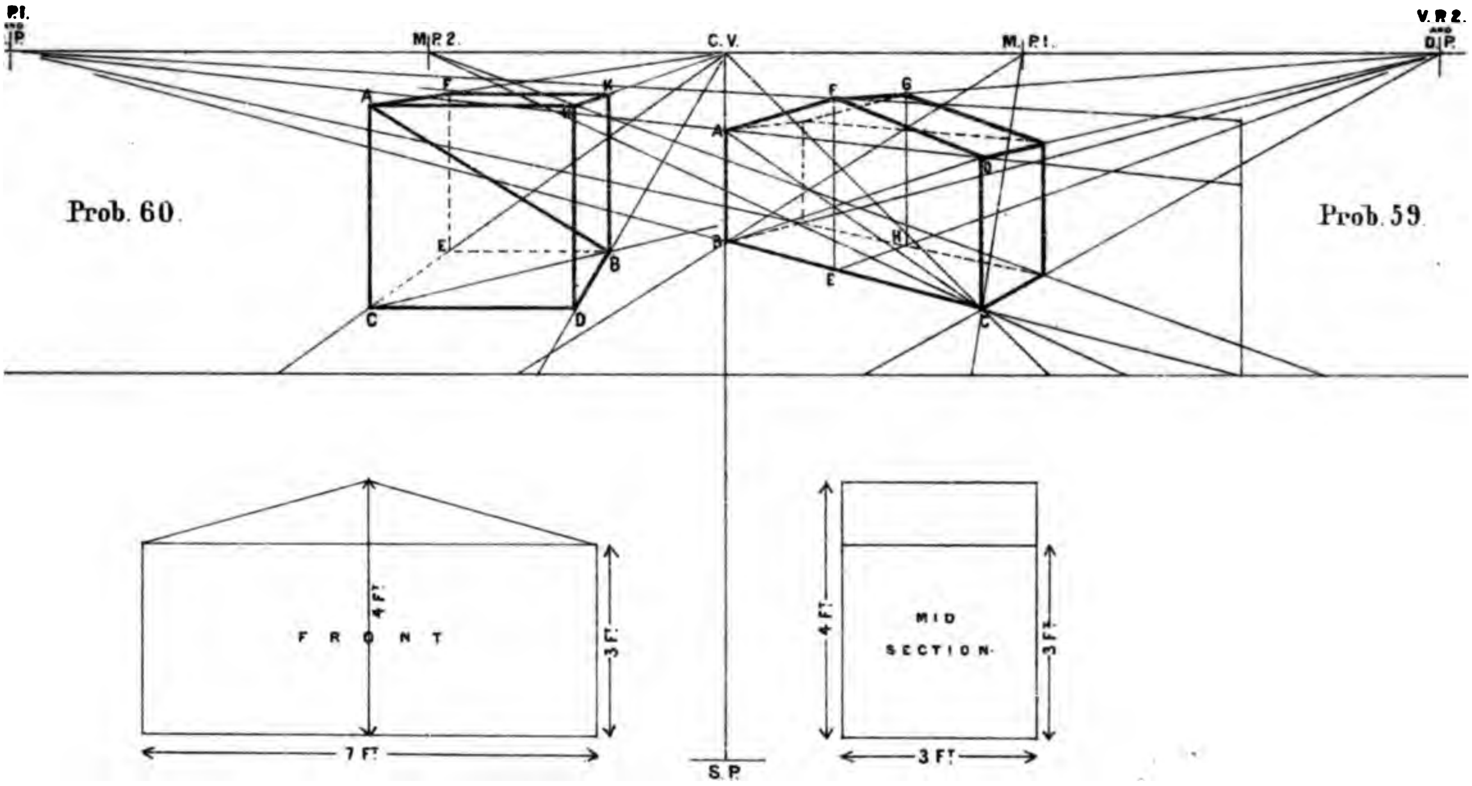
Prob. 58.

58. A cube of 4 ft. edges—nearest angle 3 ft. to the left and 2 ft. in the picture—the sides running to the right to vanish at an angle of 40° with the Picture Plane. On the top of this place an equilateral triangular pyramid 4 ft. high, so that two angles of its base shall fall on adjacent edges of the top of the cube.

Problem 59.—To obtain the rectangle E F G H, cross the diagonals of the rectangle A B C D, and through its centre thus obtained draw the perpendicular E F. From E and F draw lines to Vanishing Point 2, and on H set up the perpendicular H G.

Problem 60.—The working of this problem depends entirely on the fact that the diagonal of a square in Parallel Perspective runs to the Distance Point.

From the Centre of Vision draw a line through B to the Picture Line. From the Distance Point draw a line through B. Drop a perpendicular from A to meet the line just drawn from the Distance Point in the point C. Through C draw C D parallel to the Picture Line. From C draw a line to the Centre of Vision, and from B draw B E parallel to the Picture Line. Set up perpendiculars on D, B, and E. Draw A H parallel to the Picture Line. From A and H draw to the Centre of Vision, and join F, K. This completes the cube.



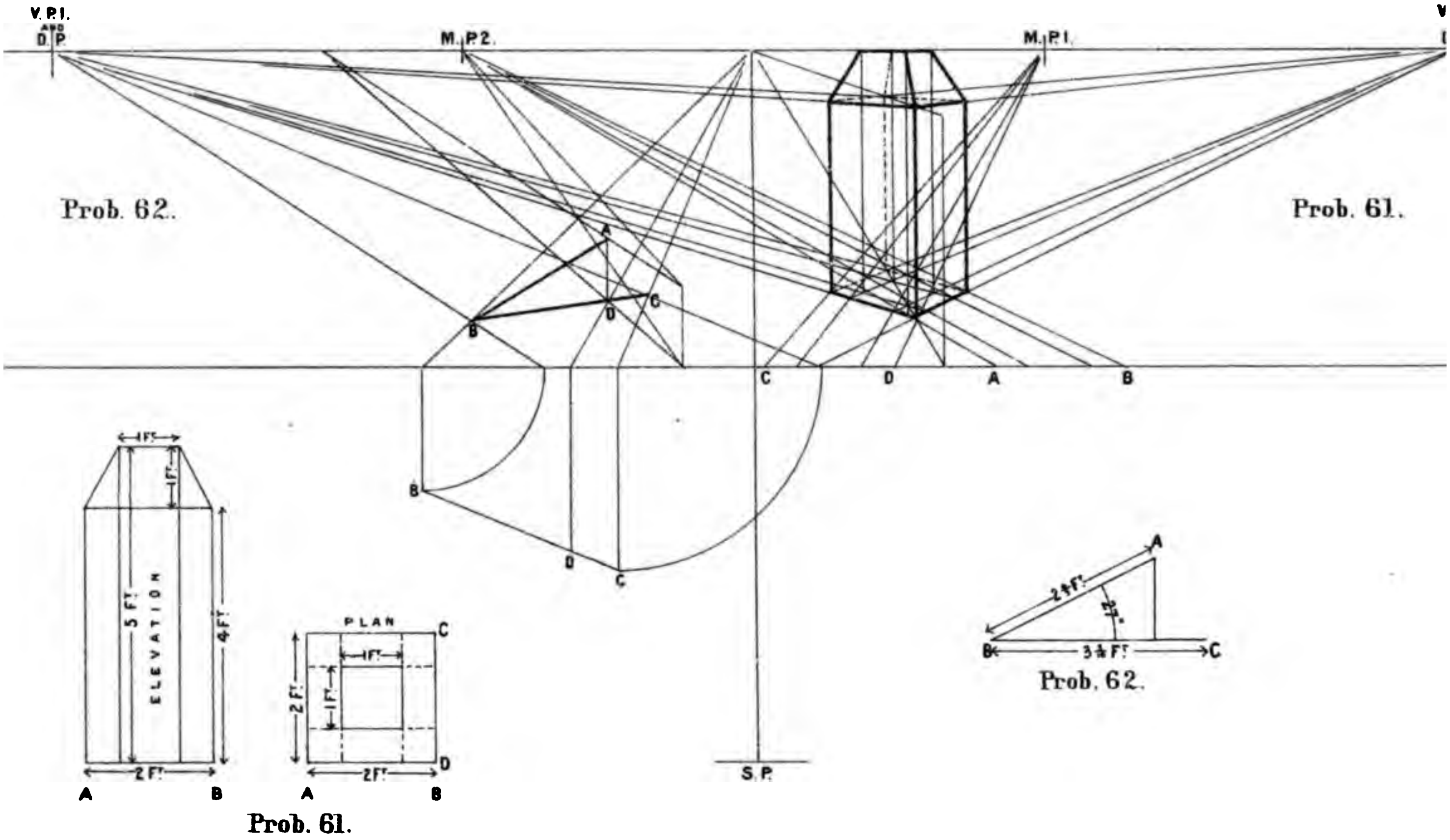
60. The given line A B is the diagonal of a cube (in Parallel Perspective), going from the left hand upper corner of the front to the right hand lower corner of the back. Complete the cube.

59. Put in Perspective the solid of which the elevation and section are given above. The nearest angle to be 5 ft. to the right and 3 ft. in the picture. The sides to vanish towards the left at angles of 45° with the Picture Plane.

Problem 61.—"Equal angles" will, of course, be angles of 45°.

As the solid is 5 ft high it just reaches the Horizontal Line, and so the top is represented by one line, since only the edges of a horizontal surface on a level with the eye would be seen.

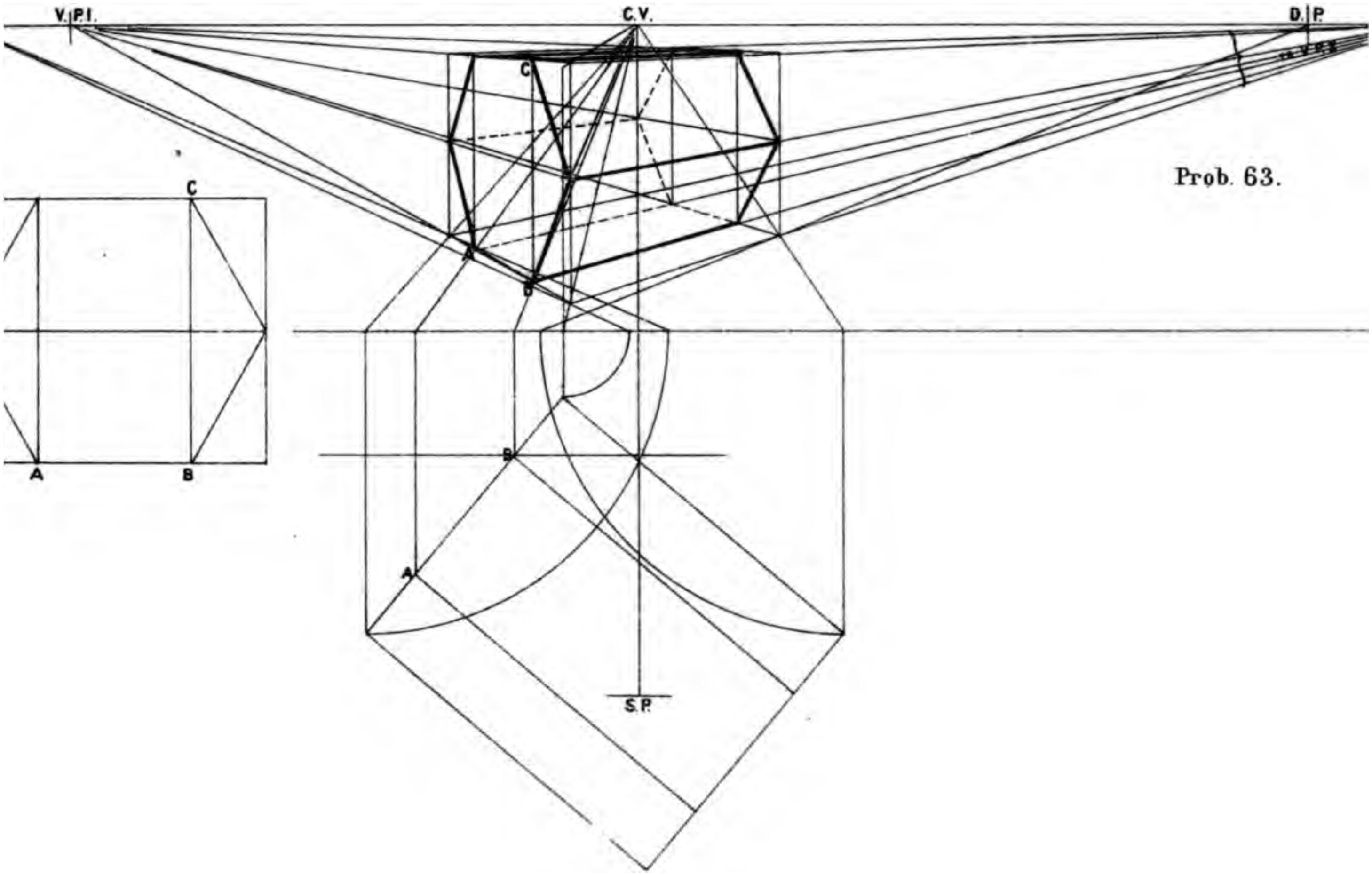
Problem 62.—This problem is somewhat like Problem 55. First drop a perpendicular from A to D. Then bring B C out of Perspective, finding on it the point D, and measure it by scale: its length is $3\frac{3}{16}$ ft. A B D is a right angled triangle. Measure the perpendicular A D: its length will be found to be $1\frac{3}{16}$ ft. Now, in any vacant space below the Picture Line, draw the line B C ($3\frac{3}{16}$ ft.) with the point D marked in it. At D set up A D ($1\frac{3}{16}$ ft.) perpendicular to B C, and join A B. We then have a geometrical representation of the angle which can be measured by the Protractor, and will be found to be 27°



62. A B C is an elevated angle. Find the length of the lines A B, B C and the size of the angle A B C.

61. The figures given above are the Plan and Elevation of a solid. Put it in Perspective, its sides making equal angles with the Picture Plane. The nearest angle to be 3 ft. to the right and 2 ft. in the picture.

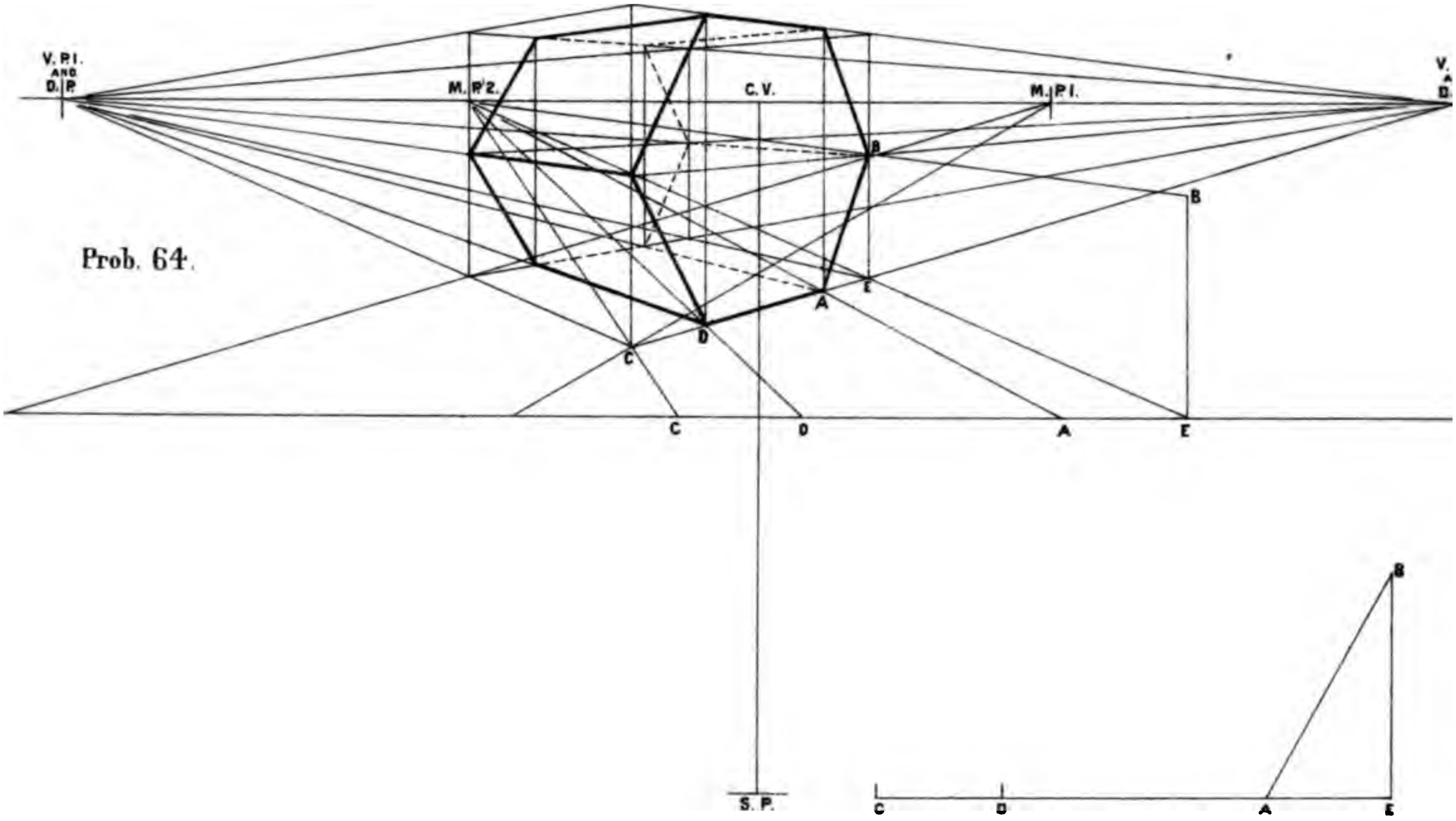
Problem 63.—If the Ground Plan and Elevation be well compared with the Perspective, the working of this problem will not be found difficult.



Prob. 63.

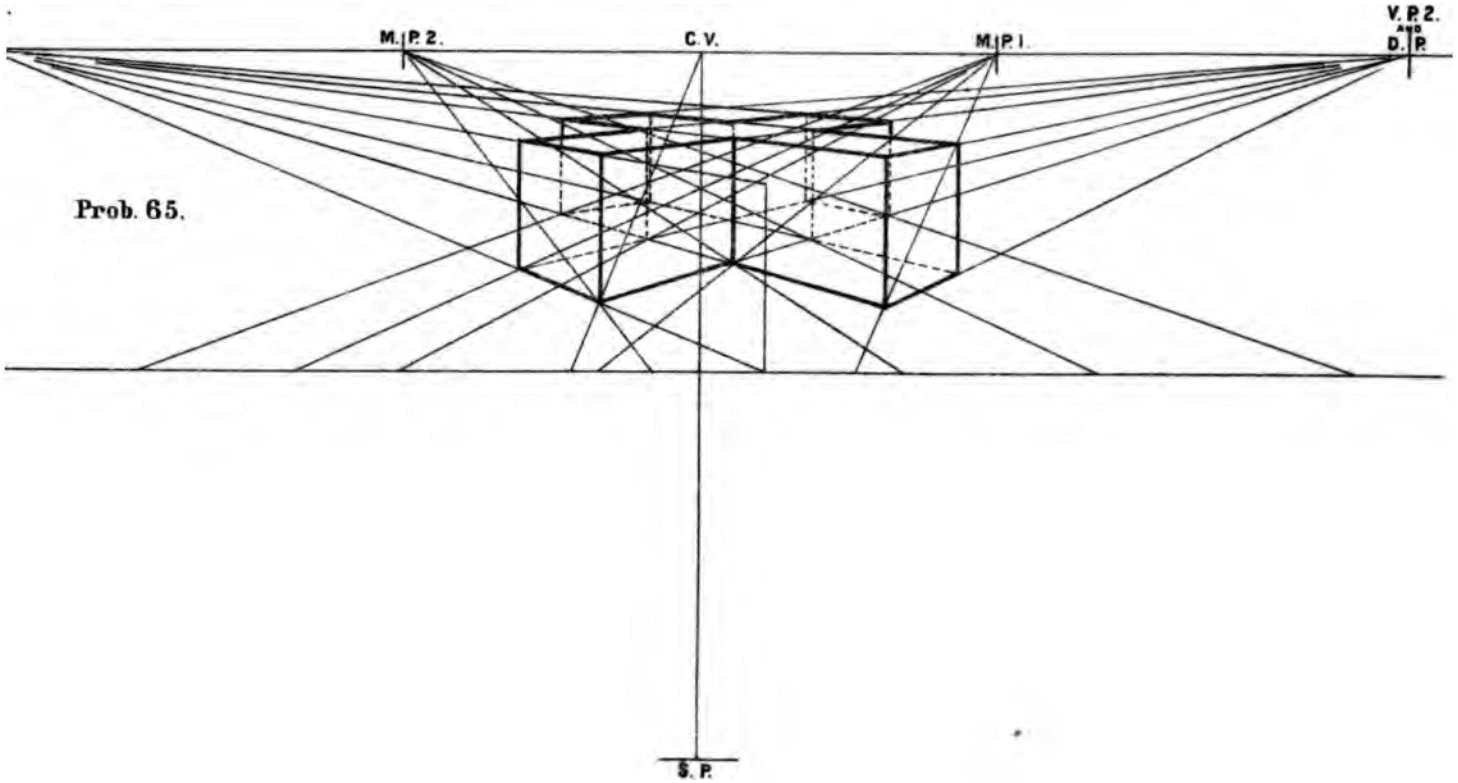
63. The figure given above is a regular hexagon. Put into Perspective a hexagonal prism 6 ft. long, so that the point B shall be 2 ft. to the left and 2 ft. in the picture, the line A B shall run to the left at an angle of 50° with the Picture Plane, and the line B C shall be perpendicular to the Ground Plane.

Problem 64.—Draw through A from the Vanishing Point 2. Drop a perpendicular B E. Draw from Measuring Point 2 through A and E to the Picture Line. Measure B E, and then construct a geometrical representation of the triangle B A E somewhere below the Picture Line. Measure on the Picture Line (from A) A D equal to A B, and D C equal to A E; then draw to Measuring Point 2, and get these points in Perspective. On C E, in Perspective, set up a rectangle double the height of B E, and draw perpendiculars across this rectangle from D and A, thus obtaining the six angular points of the hexagon forming the end of the prism. The figure is now easily completed.


Prob. 64.

64. A B is the side of a vertical hexagon in Perspective. This hexagon is the end of a prism 8 ft. long whose sides run towards the left at angles of 45° with the Picture Plane. Complete the prism.

Problem 65.—Careful observation will enable the student to understand all the work of this problem.



Prob. 65.

65. A solid cross with 4 equal arms, lying on the ground. Two arms to run at angles of 45° with the Picture Plane towards the right, the other two to cross these at right angles. Each arm to be 4 ft. long, and the ends to be 4 ft. square. Nearest angle of the arms running to the right to be 2 ft. left and 3 ft. in the picture

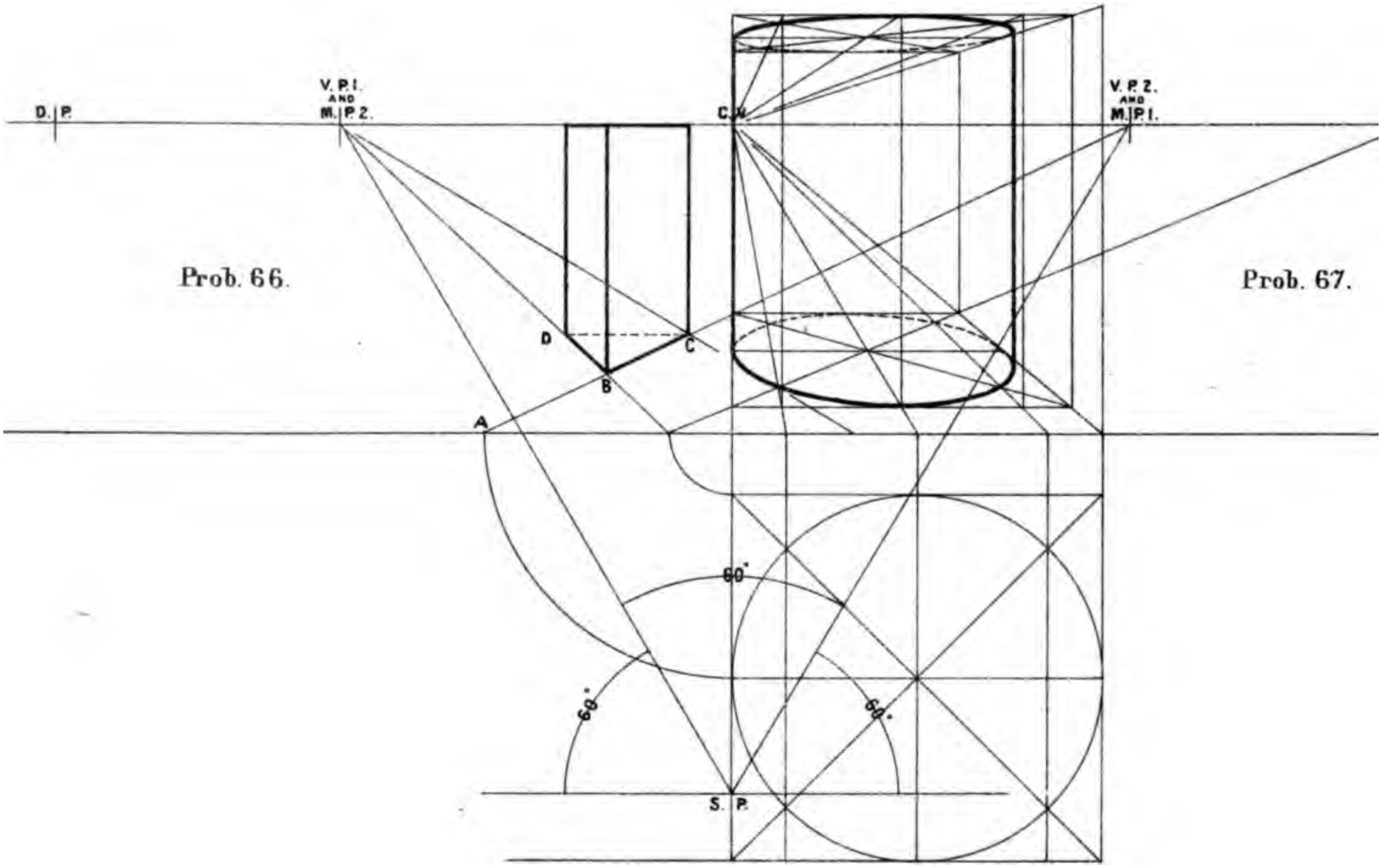
Problem 66.—The figure in this problem just reaches the horizon, and so (as in Problem 61) the top is represented by one line.

The point D is obtained by drawing through C a line parallel to the Picture Plane; for, as the other two sides of the triangle making equal angles with the Picture Plane, this third side must be parallel to it.

In this case, as in Problem 49, the Measuring Point of one line is the Vanishing Point of the other.

Problem 67.—Nothing in the working of this requires any explanation, several similar problems having been already worked.

Note.—These two problems, together with the definition of "Centre of Vision," composed the work of an Examination Paper set in Nov. 1868, and likewise in Nov. 1871.



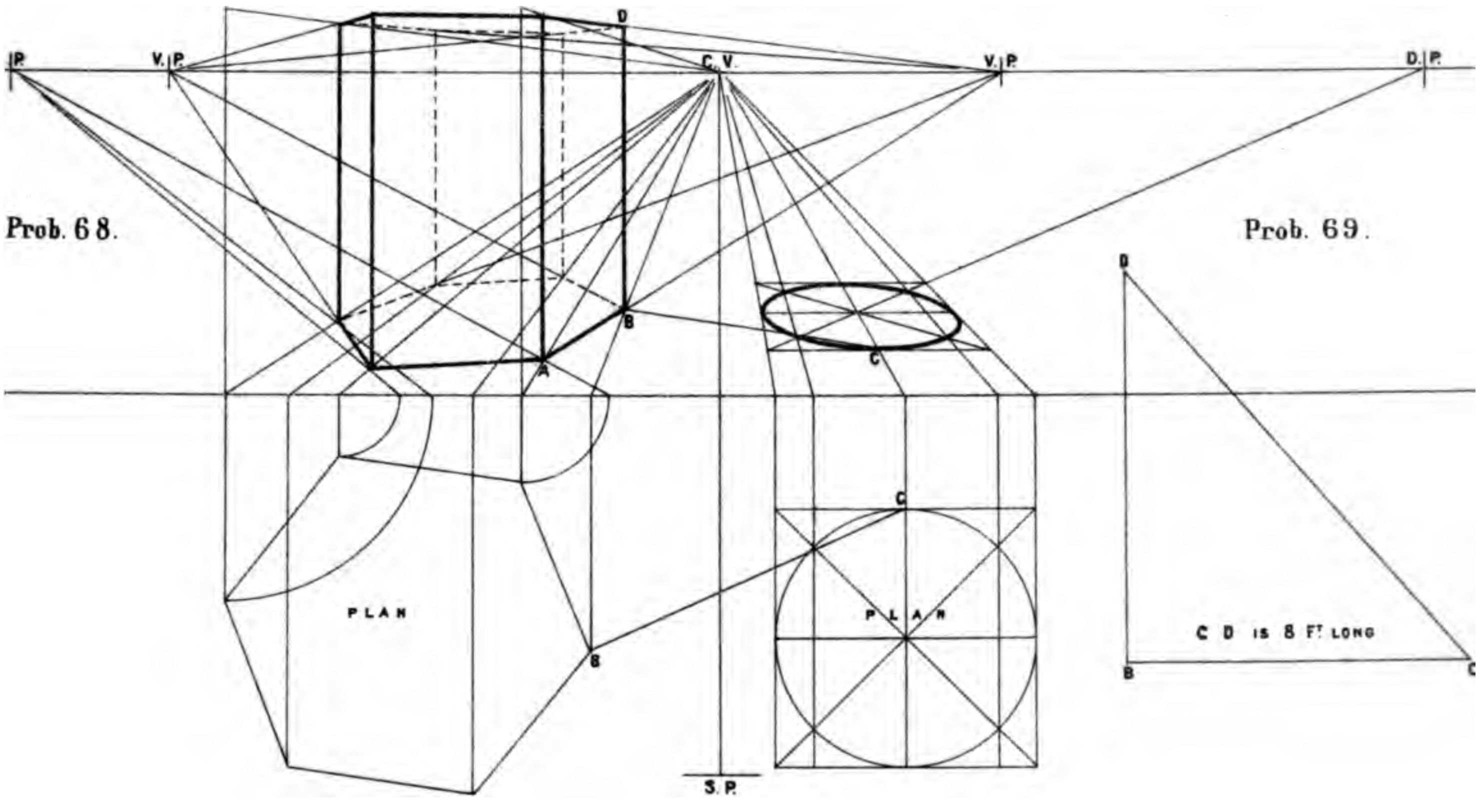
66. Find a point A on the Picture Line 4 ft. to the left. Find the Vanishing Point for lines making angles of 60° with the Picture Plane, and draw from A to this Vanishing Point —on this line find points B and C, 3 ft. and 6 ft. from A respectively. B C is one side of an equilateral triangle. Complete the triangle, and on it construct a prism 5 ft. high.

67. Place a circle in Perspective with its *centre* 3 ft. to the right and 4 ft. in the picture. *Radius* of the circle 3 ft.—on this base place a cylinder 7 ft. high.

Problem 68.—Bring the given line A B out of Perspective, and on it construct the plan of the hexagon. Put the hexagon in Perspective, and build the prism on it, in the usual way.

Problem 69.—The circle will easily be accomplished; and the measurement of the line C D is so similar to work that has already been done in Problems 55 and 62, that it needs nothing said about it here.

Note.—These two problems and the measurement of C D make another Examination Paper set in the years 1868 and 1871.



68. The line A B is the Perspective representation of one side of a regular hexagon lying on the ground. Give the Perspective representation of a vertical prism 6 ft. high, standing on this hexagon as a base.

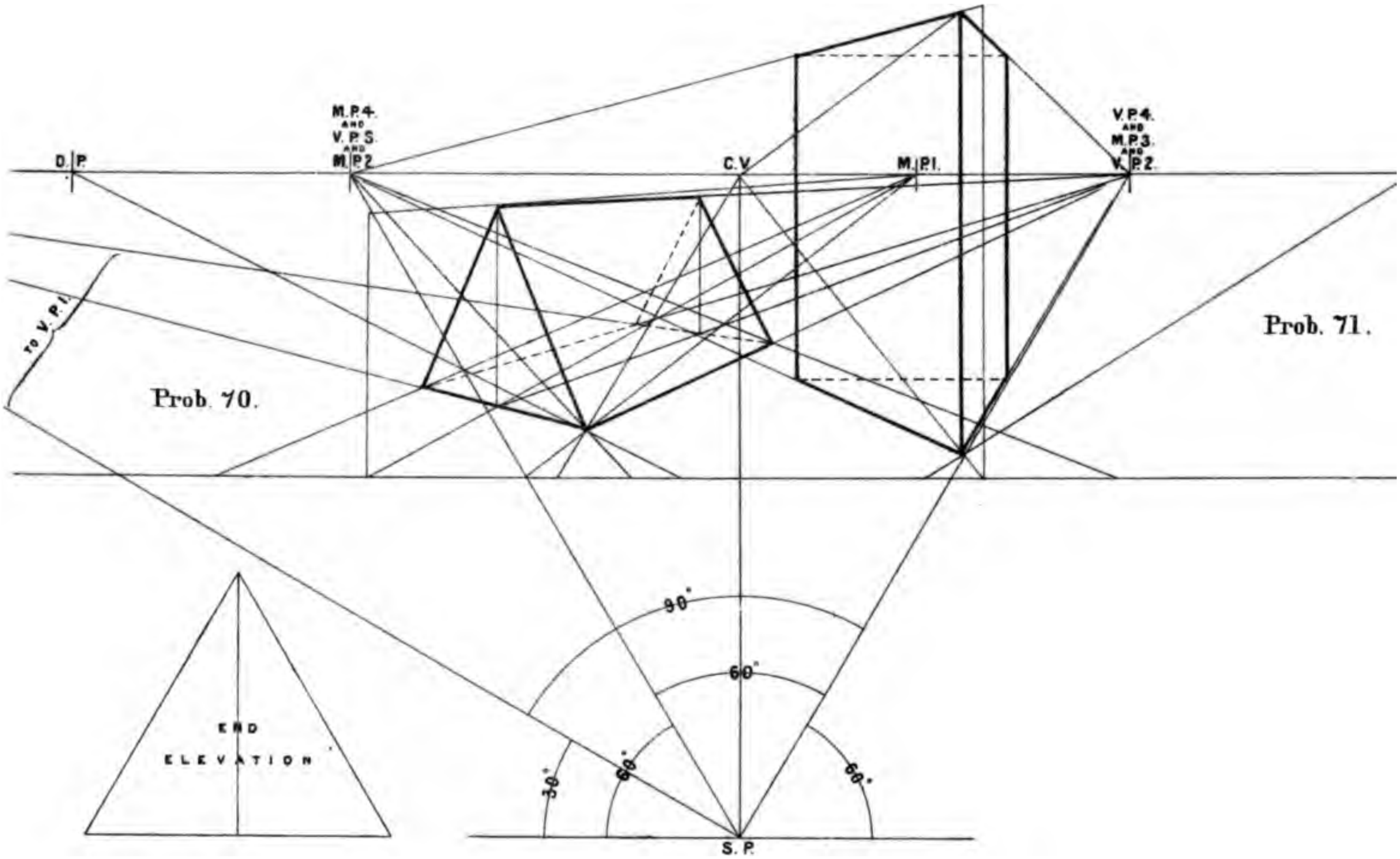
69. The circumference of a circle lying on the ground, 4 ft. in diameter, passes where it is nearest to the Picture Plane through the point C. Draw the circle in Perspective.

Give the distance by scale from the point C to the angle of the prism above B.

Problem 70.—There will be no difficulty with this problem, if preceding work be understood.

Problem 71.—That one side of the equilateral triangle may be parallel to the Picture Plane, the other two must make equal angles (60°) with that plane. This is another case in which the Vanishing Point of one line is the Measuring Point of the other.

Note.—These two problems were an Examination in 1868.

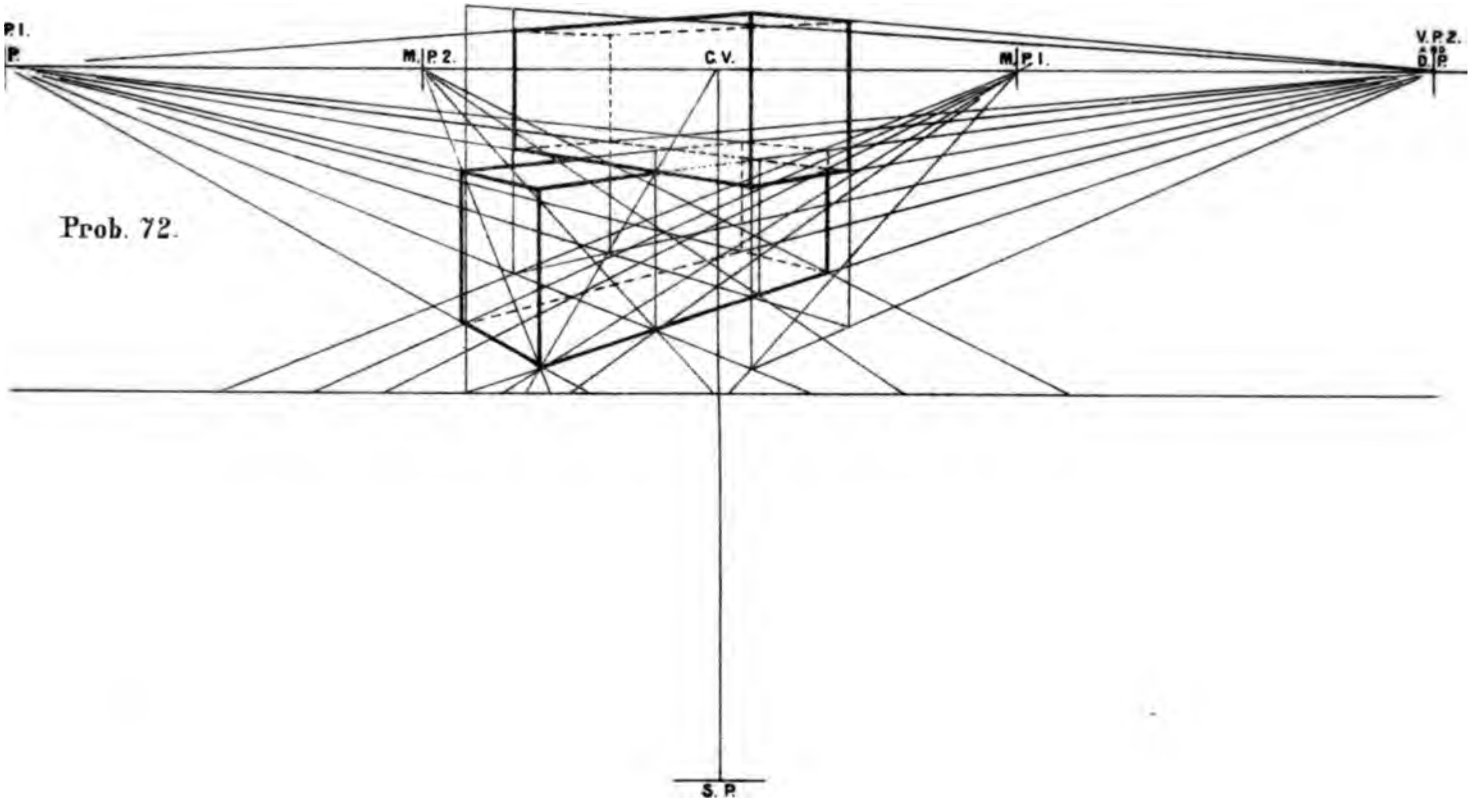


70. The base of a prism is an equilateral triangle of 5 ft. sides, its length is 8 ft. Put it into Perspective with one side lying on the Ground Plane, its longer edges being inclined to the Picture Plane at angles of 60° to the right, and the angle nearest the spectator being 3 ft. to the left and 2 ft. in the picture.

71. Put the same solid in Perspective standing on the Ground Plane in a vertical position. Two sides must be shown, and the third side must be parallel to the Picture Plane. The nearest angle to be 1 ft. in the picture and 4 ft. on the right.

Problem 72.—If the two solids are at right angles to one another, they will both make the same angles with the Picture Plane,—viz., 45°, but the long sides of the upper solid will run in the opposite direction to that in which the long sides of the lower solid run.

Note.—Another Examination Paper set in the year 1868.

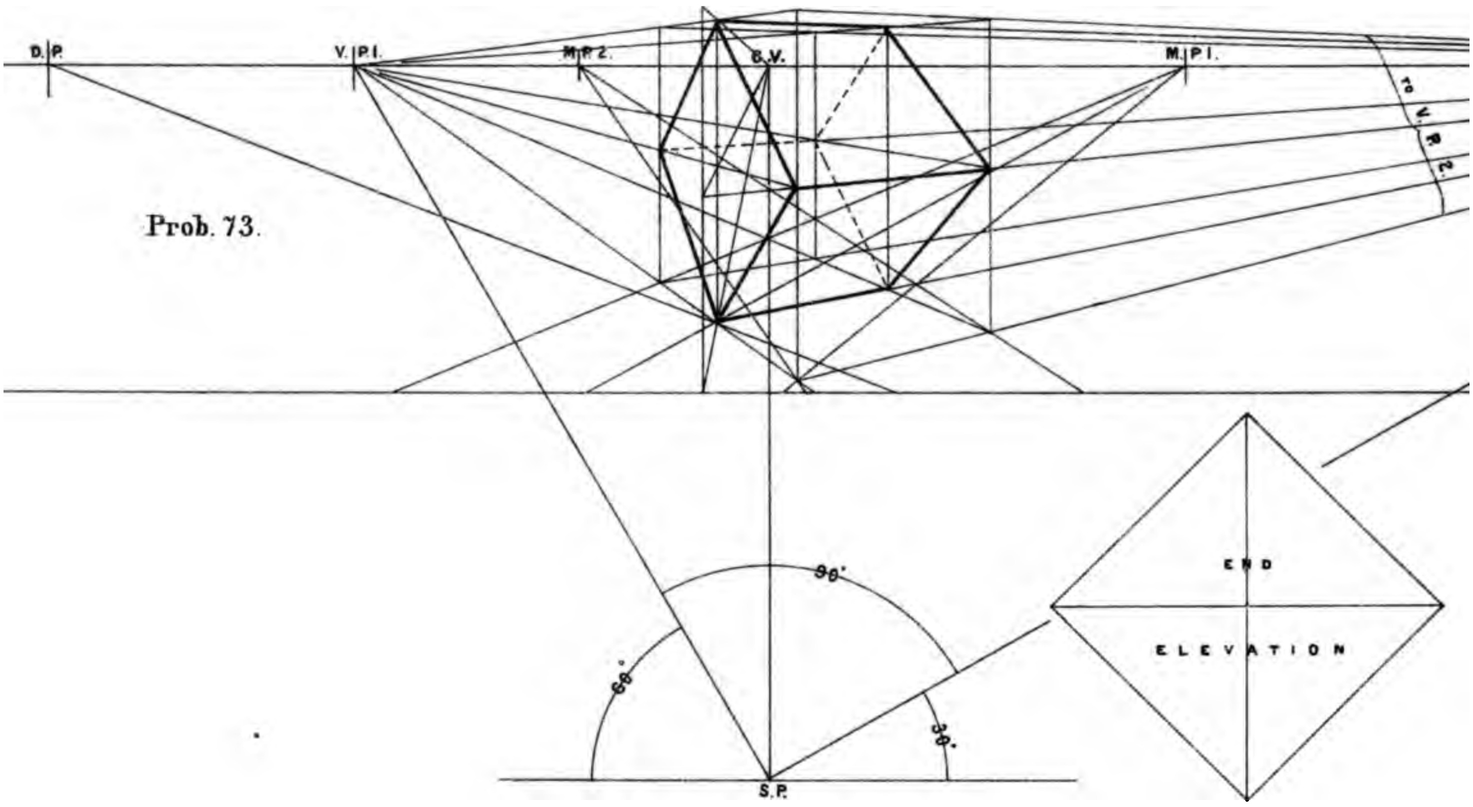


Prob. 72.

72. The base of a rectangular solid is 3 ft. square, its length 8 ft. Put it into Perspective lying on one of its 8 ft. sides, of which side the longer edges shall vanish at an angle of 45° with the Picture Plane towards the right. The nearest angle to be 3 ft. to the left and 1 ft. in the picture. Across the middle of this solid, and at right angles to it, place a similar one in a horizontal position. Let the centre of the upper solid be vertically over that of the lower one.

Problem 73.—As the cube rests on one of its edges it is clear that it will be contained in a rectangular figure 6 ft. high, 6 ft. wide, and the same length as the side of the square of which 6 ft. is the diagonal. Make a geometrical square of 6 ft. diagonals; then put into Perspective the rectangular figure in which the cube is contained, and the rest is easy.

Note.—This problem and the definition of "Picture Plane" was an Examination Paper set in 1869.



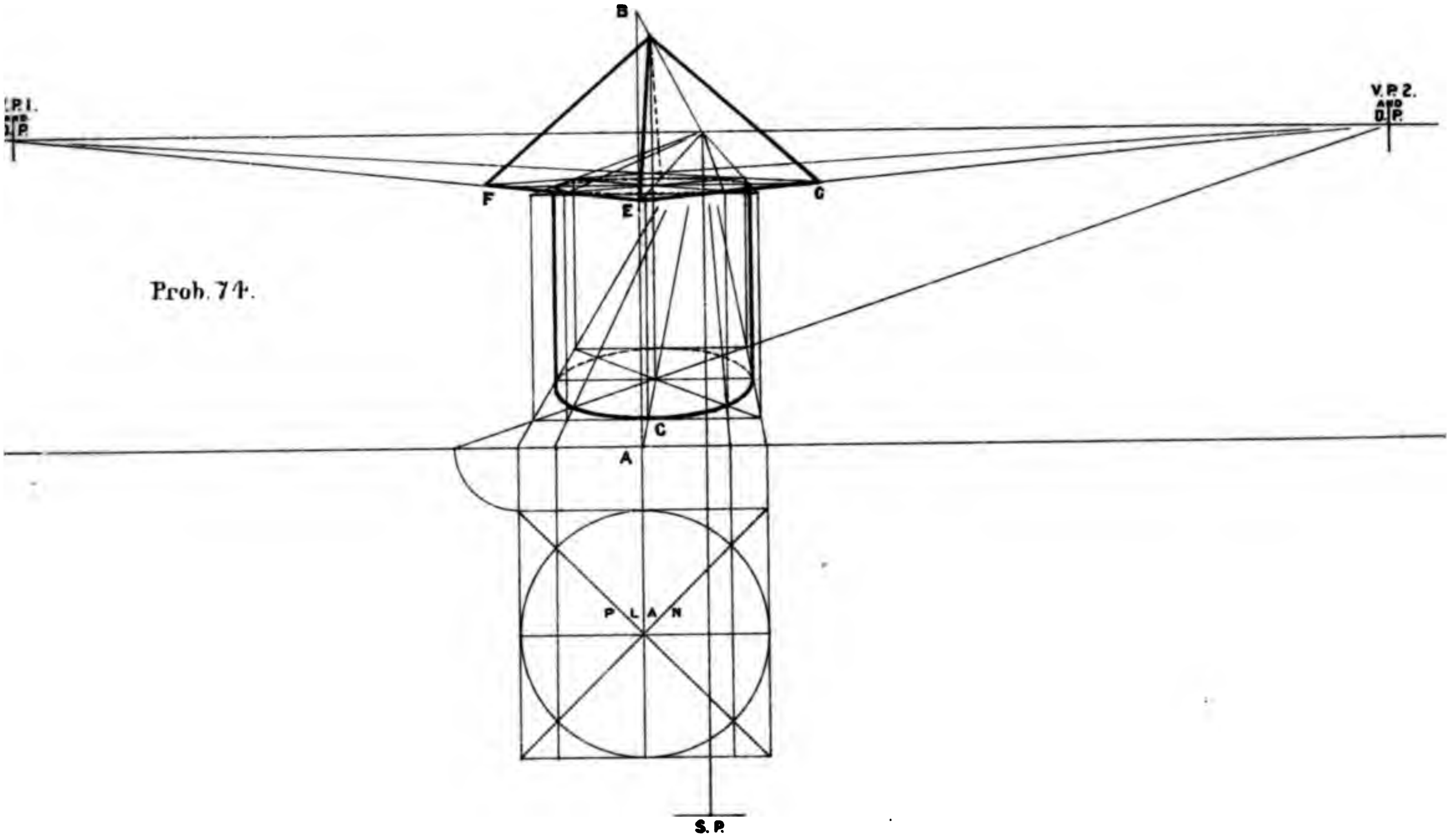
Prob. 73.

73. Find a point on the ground 3 ft. in the picture and 1 ft. left. Upon it erect a vertical line 6 ft. high. This line is a diagonal of a square of which the horizontal diagonal makes an angle of 60° with the Picture Plane towards the left Complete the Perspective representation of the cube of which this square forms a face.

Problem 74.—The *axis* of the cylinder is the line joining the centre of its top and the centre of its base. The height of both cylinder and pyramid is measured by the same Height Line A B.

"Equal angles" in the case of the pyramid will of course be angles of 45°. One diagonal of the base will therefore be parallel to the Picture Plane, the other at right angles to it. Each diagonal will go through the centre of the top of the cylinder. The nearest point of the cylinder is over C, which is 1 ft. from A (which last point is on the Picture Line). As then the pyramid is to touch the Picture Plane, the nearest angle of its base will be vertically above A, 4 ft. from the ground, *i.e.*, at E. After drawing a line from E to each Vanishing Point, the points F and G are obtained by producing the parallel diameter of the top of the cylinder to meet these lines.

Note.—This problem with the definition of a "Measuring Point" was an Examination in 1869.

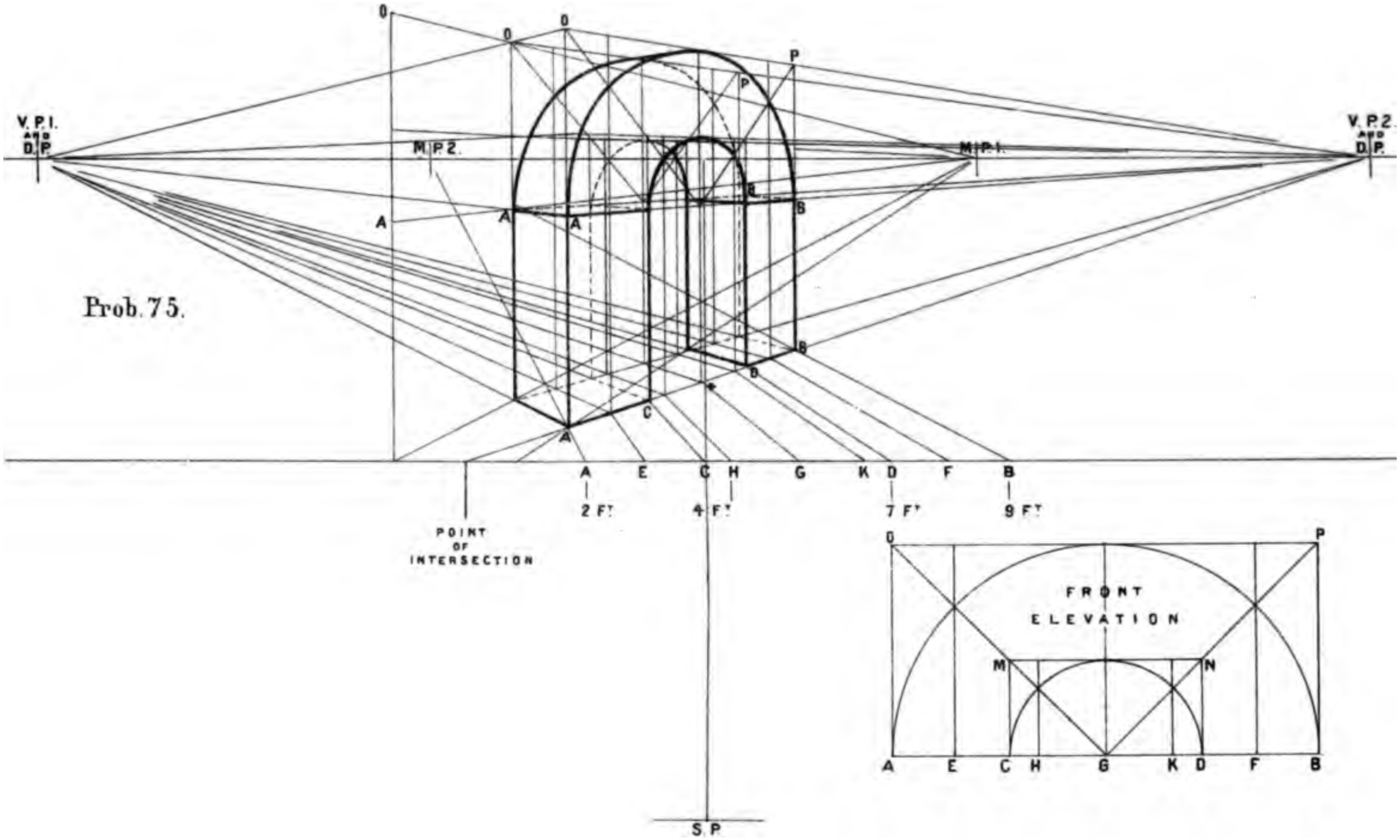


Prob. 74.

74. A right cylinder—diameter 4 ft., length 4 ft. The axis is vertical, and touches the Ground Plane 3 ft. in the picture and 1 ft. to the left. On this cylinder place a quadrangular pyramid 3 ft. high, the edges of whose square base make equal angles with the Picture Plane, and are equidistant from the cylinder. One angle of the base touches the Picture Plane.

Problem 75.—This problem requires very great care, but there is nothing in it which the student who knows how to put a rectangular solid in Perspective and understands the Perspective of the circle, cannot do.

Note.—This was an Examination Paper in 1869 and 1871.



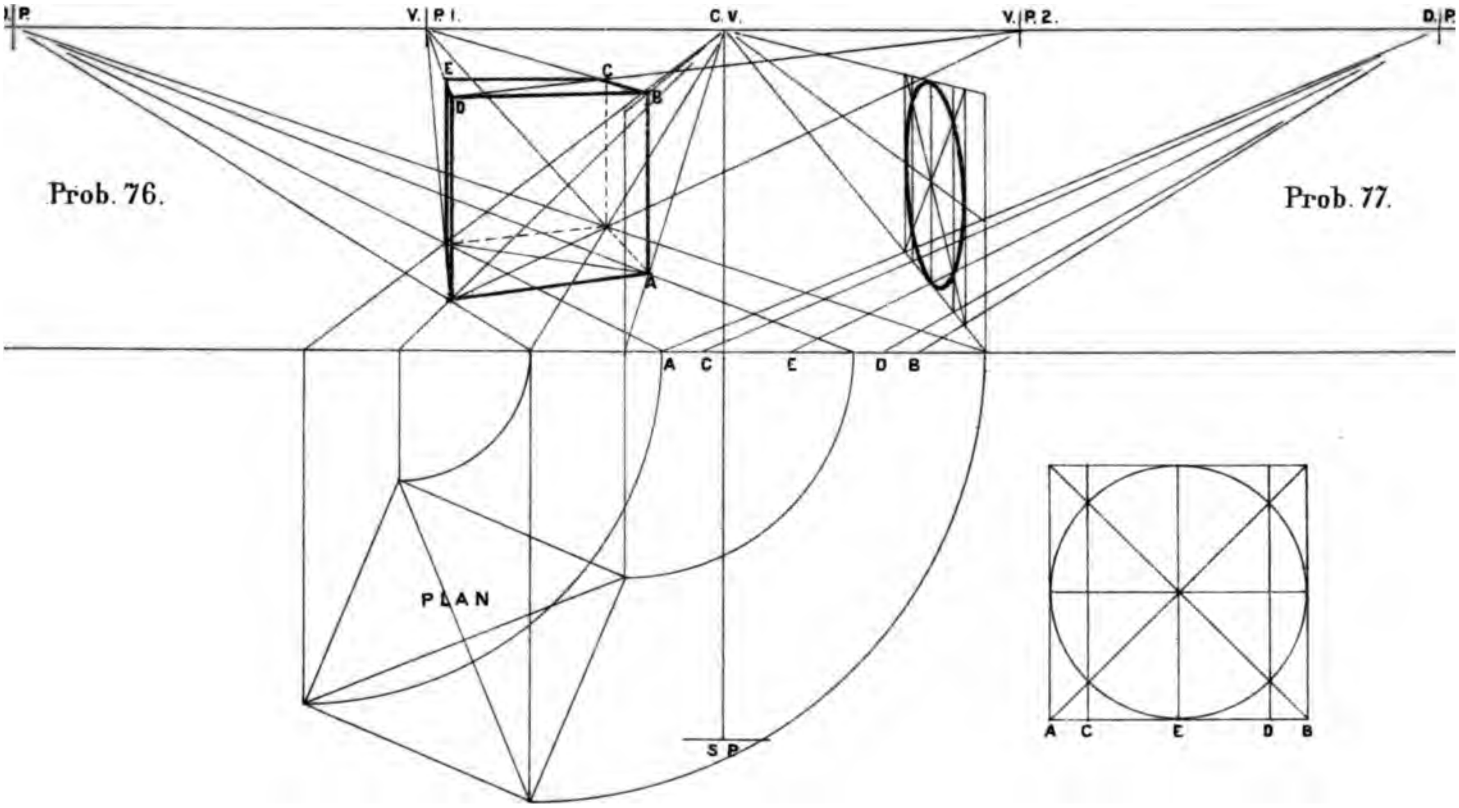
Prob. 75.

75. A horizontal line which is at an angle of 45° with the Picture Plane towards the right, intersects the Picture Line 4 ft. to the left of the spectator. Find points A, C, D, B, upon this line, respectively 2, 4, 7, 9 ft. from the point of intersection. The lines A C, D B, are near edges of two solids which are square in plan and 4 ft. high. From the top of these solids spring a semi-circular arch of the same thickness.

Problem 76.—After finding the diagonal of the base, bring it out of Perspective, and make a plan of the base, then work from this plan. In forming the top (having raised the four perpendiculars), measure the height on A B; then draw a line from B to the Vanishing Point 1; this will give point C. Draw from Vanishing Point 2 through C, and so obtain the diagonal of the top and the point D. Draw from D to Vanishing Point 1, then join E C.

Problem 77.—Similar work to this has been already done.

> *Note.*—These two problems, and another very easy one enunciated thus: "Find and mark M the Measuring Point for Horizontal Lines vanishing at 45° with the Picture Plane," composed an Examination Paper set in the years 1869 and 1871.

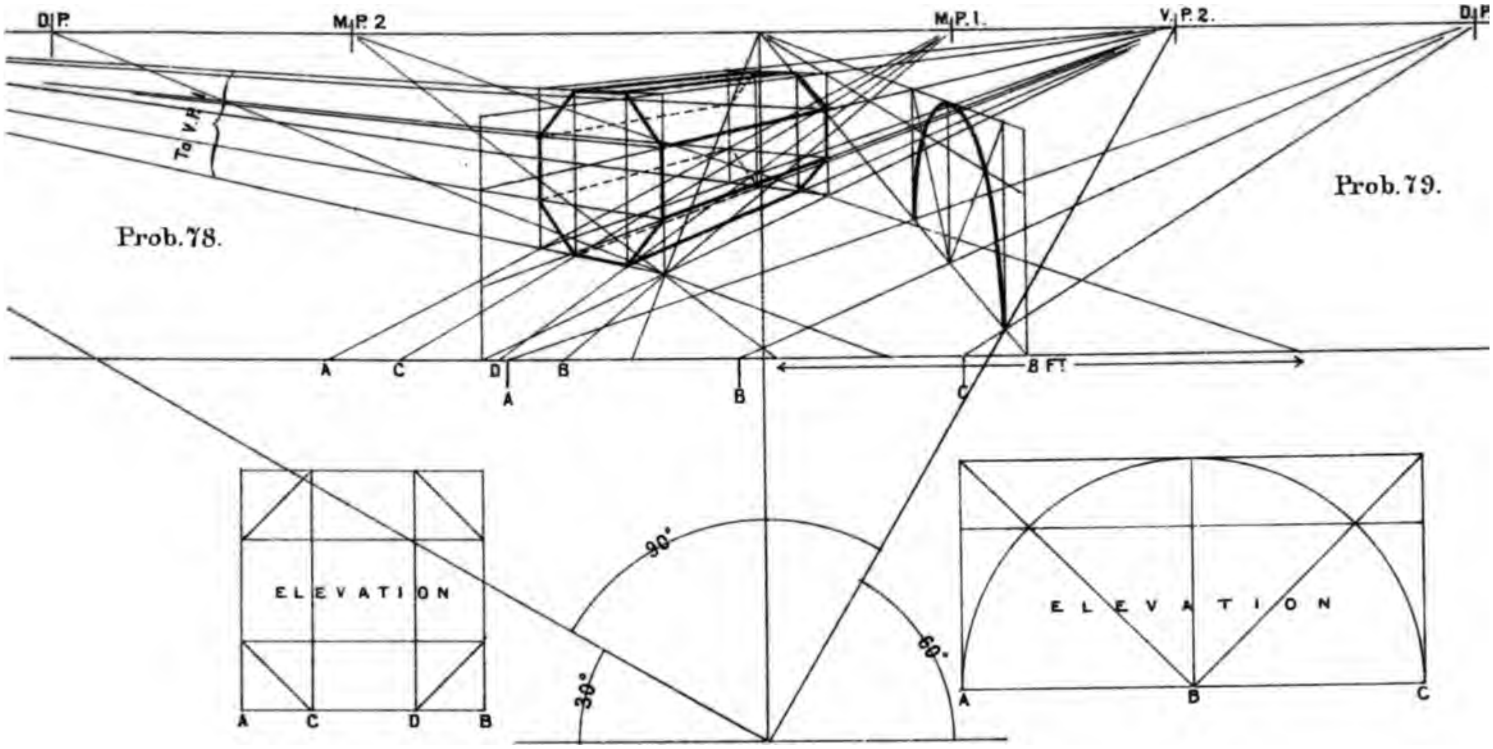


76. Find 2 points on the Ground Plane, one 3 ft. to the left and 7 ft. in the picture, the other 5 ft. to the left and 2 ft. in the picture. A right line connecting these two points is the diagonal of the base of a cube standing on the Ground Plane. Complete the cube.

77. A circle of 2 ft. radius is in a vertical plane at right angles with the Picture Plane, and touches the Ground Plane at a point 4 ft. to the right and 3 ft. in the picture. Draw its Perspective representation.

Problems 78 and 79.—Careful study will soon master these two problems.

Note.—These two were an Examination Paper in 1871.



78. An octagonal prism 8 ft. long, its base having 1½ ft. sides, and its end vanishing to the left at an angle of 30° with the Picture Plane. The nearest side of the end being 2 ft. to the left and 4 ft. in the picture.

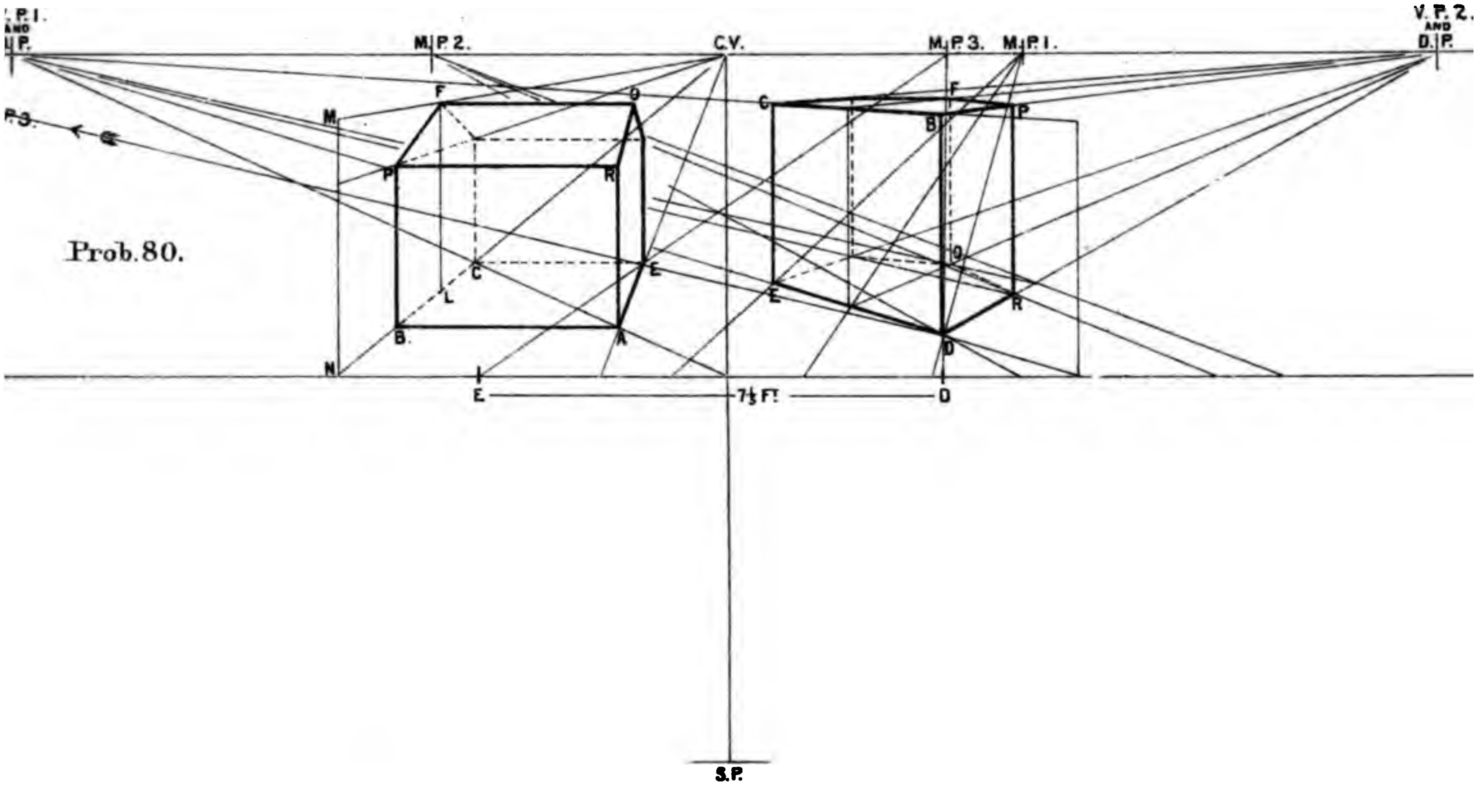
79. Find two points 4 ft. to the right; one 1 ft. in the picture, the other 8 ft. in the picture. On the line between these points erect a semi-circular vertical arch.

Problem 80.—The Height Line M N measures P B and F L. Lines from the Centre of Vision through A and B to the Picture Line give the length of A B. A line drawn from A to the Distance Point passes likewise through C, thus showing that A B C E is a square.

To measure E D, find its Vanishing Point, and then measure it by means of the Measuring Point; or bring the points E and D out of Perspective and join them, and then measure the line so obtained.

Note.—This was an Examination Paper in 1870.

Prob. 80.

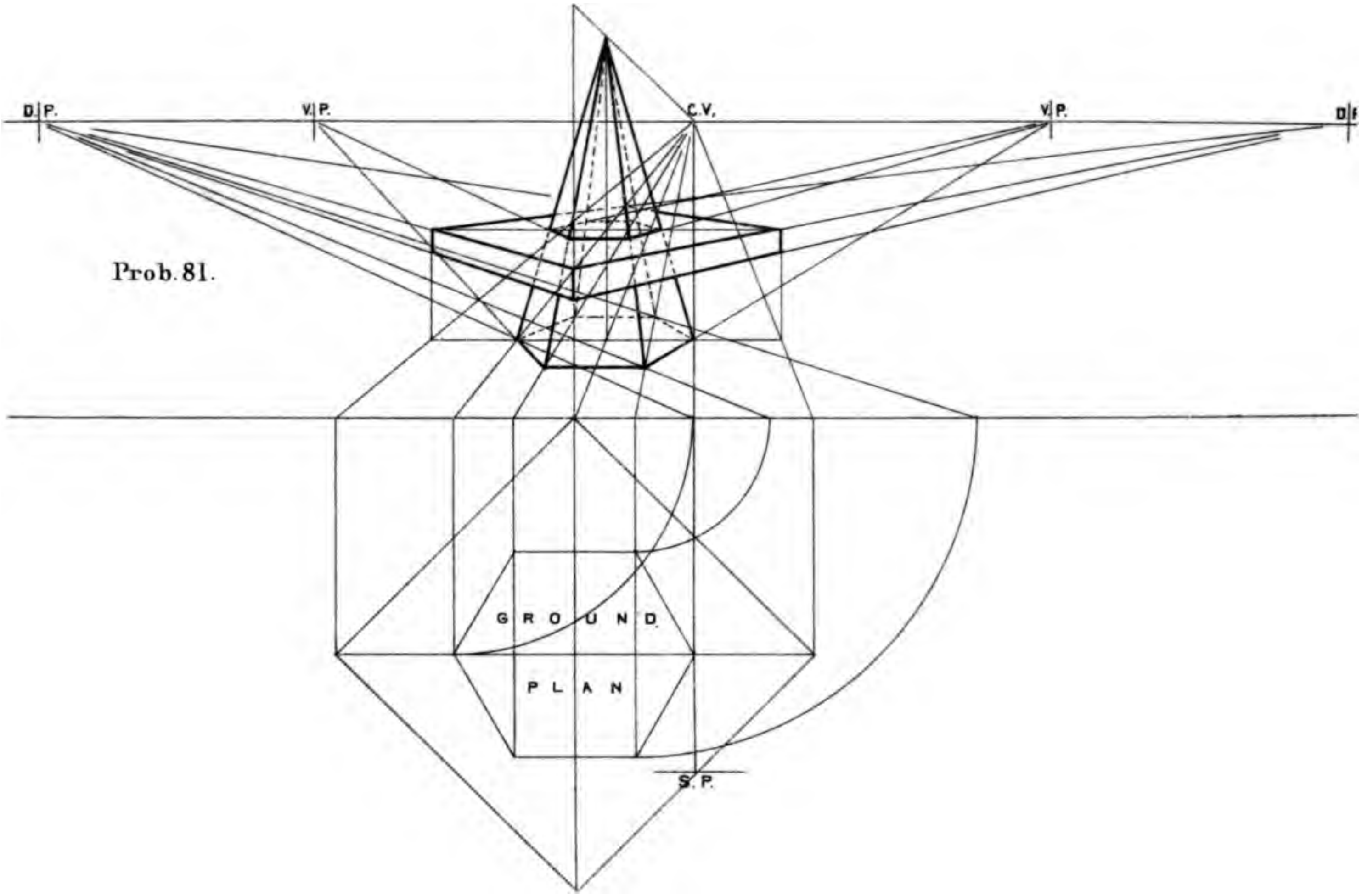


80. Find the dimensions of the solid given in Perspective on the left, and place it on the right so that point A shall be on D, and the base standing in a vertical plane which vanishes to the left at an angle of 45° with the Picture Plane. Mark off on the Picture Line the distance between E and D.

Problem 81.—Careful observation will soon enable the student to understand the working of this problem.

Note.—This was an Examination Paper about the year 1866 or 1867.

Prob. 81.



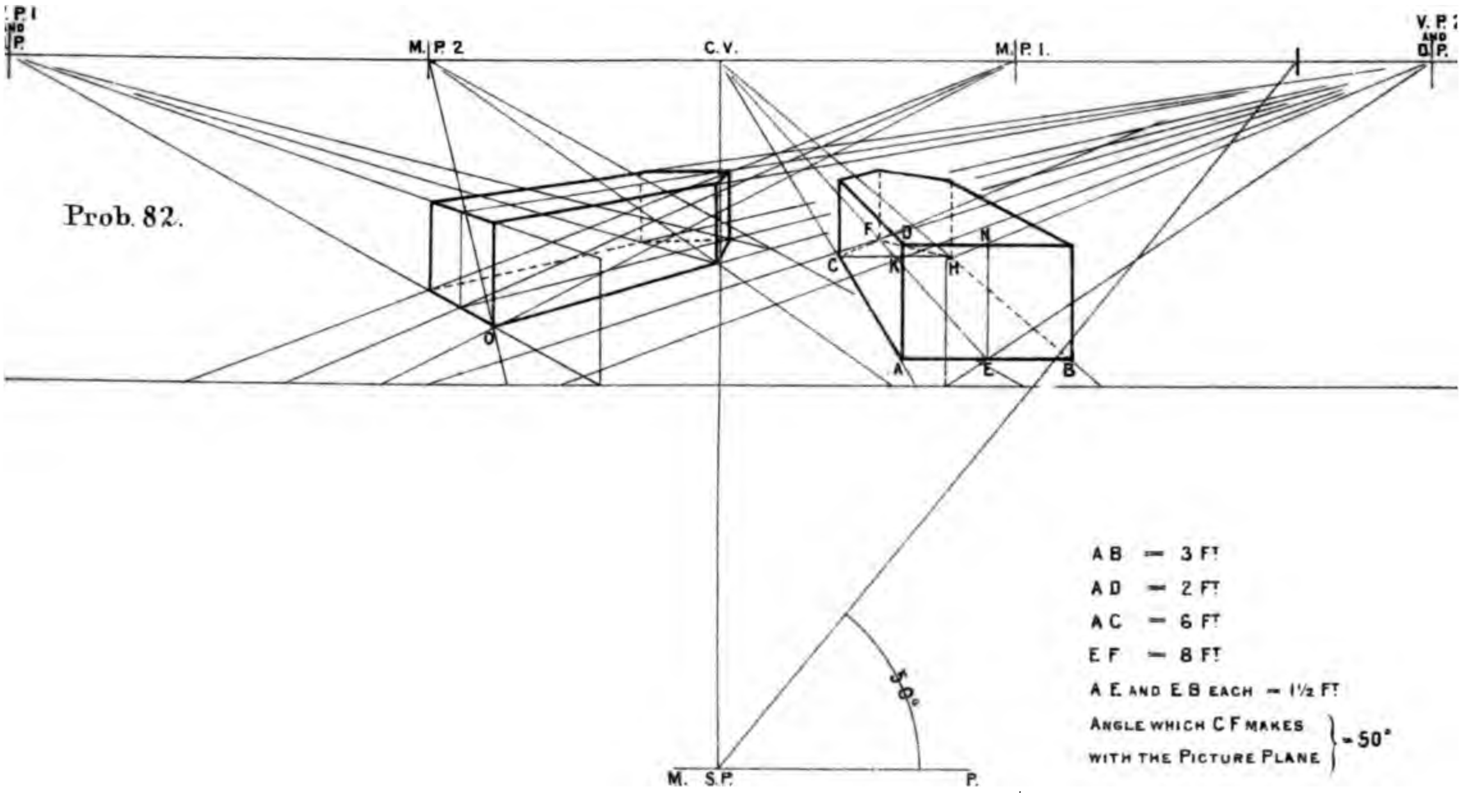
81. A hexagonal pyramid 7 ft. high, having 2 ft. sides of base, stands on the Ground Plane; the centre of its base is 4 ft. in the picture and 2 ft. to the left. Two edges of the base are parallel to the Picture Plane. A square slab of 6 inches thick is pierced through its centre by this pyramid, and rests in a horizontal position upon it at a level of 2 ft. from the ground; its long edges are at equal angles with the Picture Plane, and its nearest angle touches it.

Problem 82.—Produce A C and B H till they meet, and thus obtain the Centre of Vision. Through this draw the Horizontal Line parallel to A B. Draw the Line of Direction from the Centre of Vision at right angles to the Horizontal Line, and 11 ft. long, and so get the Station Point. Make the Base Line parallel to the Horizontal Line, and 5 ft. from it.

To find the dimensions of the solid:—Draw a line through F from the Centre of Vision. Then draw lines from the Distance Point through F, K, and E, and so get the length of the lines E K (*i. e.*, A C) and E F, on the Picture Line. Measure the height of the solid by the line E N. These are all the dimensions required for working the other part of the problem.

To find the angle which C F makes with the Picture Plane, obtain its Vanishing Point by producing it to the Horizontal Line. Then draw a line from this Vanishing Point to the Station Point, and measure the angle thus formed with the line (M P) drawn through the Station Point. This angle is the angle which C F makes with the Picture Plane.

Prob. 82.



82. The given figure is in Perspective (Parallel). Find the Centre of Vision, the Station Point, and Horizontal and Base Lines, when the distance of the spectator by scale is 11 ft., and the height of the Horizontal Line 5 ft. from the ground. Find also the dimensions of the solid, and the angle which C F makes with the Picture Plane.

Place the same solid on the left, so that the point A may be on the given point O, and the long sides run to the right at angles of 45° with the Picture Plane.

OTHER PROBLEMS TO BE WORKED.

1. Place in Perspective a hollow cylinder whose diameter is 4 ft. and depth 1 ft., so that its circular ends shall be in a vertical position and make right angles with the Picture Plane. The cylinder to be 4 ft. to the left, and to touch the Picture Plane.

2. An octagonal pyramid 8 ft. high, and having 2 ft. sides of base. The nearest side of the base to be parallel to and on the Picture Line 4 ft. to the right.

3. An equilateral triangular prism 9 ft. long, whose triangular ends have 5 ft. sides. The long sides to run to the left at an angle of 42° with the Picture Plane, and the nearest angle of the base to be 3 ft. to the left and 1 ft. in the picture.

4. A block of stone 5 ft. square and 2 ft. thick, lying on one of its square sides; the other sides to make equal angles with the Picture Plane; and the nearest corner to be 2 ft. to the right and 1 ft. in the picture. On the top of this block, centrally, stand a pyramid 6 ft. high, having a base 5 ft. square, two sides of which are to be parallel to the Picture Plane.

5. A hexagonal block 2 ft. high, its hexagonal ends having 3 ft. sides, and two of its sides being parallel to the Picture Plane. Stand centrally on this a hexagonal prism 7 ft. high, whose ends shall have 1½ ft. sides. The sides of the prism to be parallel to the sides of the block.

6. A cone 5 ft. high, on a base 3 ft. in diameter, touching the Picture Line 2 ft. to the right. Let this cone pierce a square slab 1 ft. thick through its centre. The slab tc rest horizontally on the cone 3 ft. from the ground, its sides to make angles of 45° with the Picture Plane, and its nearest corner to touch it.

7. A slab 1 ft. high and 6 ft. square, making equal angles with the Picture Plane, and touching it 2 ft. to the left. Stand centrally on this slab an ordinary bowl, the top of which shall be 5 ft. in diameter, and the bottom 3 ft.

8. A hexagonal block 2 ft. thick, its hexagonal base having 4 ft. sides. One side of base to run to the right at an angle of 40° with the Picture Plane. The nearest angle to be 3 ft. to the left, and 3 ft. in the picture. On the top of this, centrally, place another smaller hexagonal block of the same thickness, whose sides shall be parallel to the sides of the lower block, and 1 ft. away from them.

9. A flagstaff, 10 ft. high, 27 ft. to the right, and 30 ft. in the picture.

10. A block 8 ft. square and 2 ft. high. One side to run to the right at an angle of 40° with the Picture Plane. The nearest angle to be 2 ft. to the left and 1 ft. in the picture. Place a cylindrical column 7 ft. high and 5 ft. in diameter, upright and centrally on this block.

11. A rectangular solid 6 ft. high, having base 3 ft. square. One side of the base to run to the right at an angle of 50° with the Picture Plane. The nearest angle to be 3 ft. to the left and 4 ft. in the picture. Through the centre of this solid let a hexagonal prism 9 ft. long run towards the right, whose ends shall have 1½ ft. sides.

12. A millstone 6 ft. in diameter and 1½ ft. thick, lying on the ground, and touching the Picture Plane 4 ft. to the right. Through its centre pierce a hole 1½ ft. square.

13. The same millstone in a vertical position, its circular ends making right angles with the Picture Plane. The stone to touch the Picture Plane 4 ft. to the left. The same size hole through its centre.

14. A circular block 7 ft. in diameter and 2 ft. thick, resting on one of its circular ends, and touching the Picture Plane 4 ft. to the right. On the centre of this stand an octagonal prism 6 ft. high, whose octagonal ends shall have 1½ ft sides, and one of whose rectangular faces shall be parallel to the Picture Plane.

15. A cube of 4 ft. with one face making an angle of 50° towards the right; its nearest angle being 3 ft. to the left and 2 ft. in the picture. Describe a circle on each visible face 4 ft. in diameter.

16. A regular heptagon of 2 ft. sides lying on the ground, with one side making an angle of 40° with the Picture Plane towards the left; its nearest angle being 2 ft. to the left and 1 ft. in the picture. On this base erect a prism 8 ft. high.

17. A pentagonal prism 6 ft. long lying on one of its sides, which vanish towards the right at an angle of 40° with the Picture Plane. The sides of the pentagonal ends to be 2 ft. long.

18. A cube of 3 ft. with its sides making equal angles with the Picture Plane; its nearest angle being 2 ft. to the right and 2 ft. in the picture. On it stand a cone 5 ft. high, whose base is 3 ft. in diameter.

19. Find two points A and B on the ground, 2 ft. to the left and 3 ft. and 8 ft. respectively in the picture. A B is one side of a vertical square; complete the square, and in it inscribe a circle 5 ft. in diameter. This circle is one end of a cylinder 6 ft. long, lying on the ground parallel to the Picture Plane; complete the cylinder.

20. A hut, floor 8 ft. long and 5 ft. wide, walls 7 ft. high, ridge of roof 8 ft. high, the long sides to run to the right at angles of 45° with the Picture Plane, the nearest corner to be 1 ft. to the left and 2 ft. in the picture. Place in the middle of the end a doorway 5 ft. high and 2½ ft. wide.

21. A rectangular solid 3 ft. high on a base 4 ft. square. Place centrally on this a circular stone 8 ft. in diameter and 2 ft. thick. The nearest angle of the solid to be 3 ft. to the right and 1 ft. in the picture.

22. A semi-circular arch composed of stone 2 ft. square. The outer diameter to be 8 ft. Let it run towards the right at angles of 45° with the Picture Plane, and let the nearest point of it be 1 ft. to the left and 2 ft. in the picture.

23. A plank 5 ft. square and 2 ft. thick in a vertical position, making angles of 60° with the Picture Plane towards the right, its nearest angle being 3 ft. to the left and 3 ft. in the picture. Let a cylindrical solid 3 ft. in diameter pass through the centre of this plank, so that the nearest point of its end shall touch the Picture Plane,

24. A regular hexagon in a vertical position, composed of bars 1 ft. thick, and making angles of 45° with the Picture Plane. The nearest angle to touch the Picture Plane, and the outside edges to be 2½ ft. long.

25. A cube of 4 ft. edges with its sides making equal angles with the Picture Plane; its nearest angle being 2 ft. to the left and 2 ft. in the picture. On top of this place (centrally) another cube of the same size, having two of its sides parallel to the Picture Plane.

26. A slab 6 ft. square and 1 ft. thick lying horizontally on the ground, with one of its sides making an angle of 50° with the Picture Plane towards the right, its nearest angle touching the Picture Plane 3 ft. on the right. On the centre of this slab, and parallel to it, stand a solid cross 6 ft. high, composed of material 2 ft. square; the arms of the cross to be 2 ft. long, 2 ft. from the top of the shaft, and to run towards the right.

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